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On the zoology of 2d $\mathcal{N} = (0, 2)$ dualities gauge theories with antisymmetric matter

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ABSTRACT: In this paper we investigate and propose new dualities involving 2d gauge theories with $\mathcal{N} = (0, 2)$ supersymmetry. In the first part of the paper we focus on $\mathrm{SU}(n)$ gauge theories with two antisymmetric chirals. The gauge theories are non-anomalous if we consider, in addition to such matter content, n_f fundamental and n_a antifundamental chirals, provided the constraint $n_f + n_a = 4$. By exploring the five possibile scenarios arising from this constraint we provide in each case evidences of a dual LG description, by matching the 't Hooft anomalies and deriving the relation between the elliptic genera in terms of other *more fundamental* dualities. In the second part of the paper we provide a 4d origin for a gauge/LG duality already stated in the literature, that does not descend from any known s-confining duality. In the last part of the paper we focus on dualities for $\mathrm{SU}(n)$ and $\mathrm{USp}(2n)$ models with antisymmetric Fermi multiplets, obtained from dimensional reduction of 4d parent dualities.

Contents

1 Introduction

The existence of IR dualities is a fascinating aspect of quantum field theories, because it opens up the possibility of alternative field theoretical descriptions of the same physical setup. Ubiquitous examples have been obtained in the supersymmetric setup, where dualities played a prominent role in different dimensions and degree of supersymmetry. An ultimate goal would be achieving a connection with the real world by uncovering dualities beyond the supersymmetric case. For this reason it is useful to take an intermediate step by finding dualities with a low amount of supercharges. While in 4d the lowest possible cases require four supercharges, in lower dimensional physics the number of supersymmetry generators can be lowered. An interesting setup involves 2d dualities with $\mathcal{N} = (0, 2)$ supersymmetry, as in this case a whole class of dualities can be achieved by dimensional reduction of 4d Seiberg-like dualities from a twisted compactification on a two sphere.

Typically, such compactification of 4d dual pairs results in relations between collection of theories in lower dimension, obscuring the possible claims on pure 2d dualities. However, in [1], elaborating on the results of localization, the authors discuss a prescription that allows to infer the existence of 2d dualities starting from 4d ones. The prescription requires to fix the R charges of the 4d fields to be non-negative integers. In this way the whole contribution to the 4d $S^2 \times T^2$ topologically twisted index [2] comes from the zero flux sector only, discarding the sum over the remaining ones. By translating the same constraints imposed on the electric side to the magnetic one via the 4d duality map, one obtains a candidate 2d $\mathcal{N} = (0, 2)$ duality. The 't Hooft anomaly matching are automatically satisfied and the equivalence of the elliptic genera is a consequence of the equivalence of the 4d topologically twisted indices.

In principle one cannot trust such dualities without any hesitation and further checks are necessary. For example the theories obtained in this way usually have non-compact direction in the target space. For this reason the c-extremization procedure of [3, 4] requires some care. Furthermore, the matching of the elliptic genera in this case is reliable only in the massive case.

Remarkable checks of such dualities have been performed in the literature when one restricts to the s-confining limit of Seiberg duality [5] and Intriligator-Pouliot duality [6]. In such cases the dual theories correspond to Landau-Ginzburg (LG) models. Furthermore, as observed in [1], in the USp$(2n)$ case the prescription described so far does not apply to the reduction of the Intriligator-Pouliot beyond the s-confining regime, whereas this is allowed for Seiberg duality.

Restricting to the reduction of s-confining gauge theories, it has been shown in [7–9] that the other 4d s-confining dualities reduce to 2d gauge/LG dualities that can be derived in a pure 2d picture by generalizing the tensor deconfinement technique of [10, 11]. This approach of deriving dualities from dualities was used recently in various dimensions [12–24] in order to show that several dualities in presence of tensor matter are consequence of more *basic* ones with only fundamental matter. A relevant result for our discussion was obtained in [25], where it was shown that the classification of

[26] for SU(N) and USp($2n$) dualities can be obtained using only two *basic* s-confining dualities, i.e. the s-confining limits of Seiberg and Intriligator-Pouliot dualities. The same phenomenon has been generalized in [9] for 2d dualities obtained by applying the prescription of [1] to such s-confining cases[1].

Furthermore, the idea of deriving 2d dualities with tensorial matter in terms of *basic* gauge/LG dualities can be applied to other cases, where a 4d parent s-confining description is absent. Some examples have already been studied in [8, 9], but more cases are expected. A reason behind such expectations is that the constraints imposed by gauge anomalies are milder in 2d than in 4d. In addition, the similarities between the dualities found in [8, 9] and the 3d ones with a similar field content obtained in [15, 19, 22, 27] suggest that 2d gauge/LG dualities should exist also for SU(N) gauge theories with two antisymmetric chirals.

Motivated by this last observation, in this paper we continue our investigations of 2d $\mathcal{N} = (0, 2)$ SUSY QFTs dual to LG models, providing a generalization of the previous results found in [9]. We mostly focus on SU(N) gauge theories with antisymmetric and fundamental matter, considering both chiral and Fermi fields.

The first class of models under investigation consists of gauge/LG dualities without a known 4d origin, *i.e.* no 4d confining dualities available in the literature give rise to the 2d dualities argued here through a topological twist. We discuss SU(N) gauge theories with two antisymmetric, n_f fundamental and n_a antifundamental chirals, such that $n_f + n_a = 4$. Notice that, even if such dualities do not descend from any 4d parent, they share a similarity with analogous 3d dualities proposed in [27], and further checked in [22, 28]. Such relation agrees with the results of [9], where the various 2d dualities were shown to be in line, even in absence of a 4d parent, with analogous 3d dualities. The main checks of the dualities proposed here consist of 't Hooft anomaly matching and of the study of the elliptic genus. While the first check is straightforward, the matching of the elliptic genera in this case cannot be inferred from the one of any 4d partition function, and its analysis is a crucial test of the duality. Here we show that the matching of the elliptic genera reduces to the one of simpler dualities, that hold in absence of any tensorial matter field and that have a 4d origin. The analysis is very similar to the one performed in [7–9] and it requires a duality that is not completely under control in 2d. The auxiliary duality involves an USp($2n$) gauge theory with $2N + 3$ fundamental chirals and 1 fundamental Fermi. This model is dual to an LG, provided that a symmetry of the electric theory is obstructed[2]. Such obstruction is due

[1]The sporadic cases have not been studied in detail in [9], but the same result is expected to hold for them as well.

[2]We will often refer, with a slight abuse of notation, to the electric and the magnetic phase for the 2d models under investigation, in analogy, when it exists, with the 4d counterpart.

to the choice of R charges used to perform the twist on the two-sphere. As observed in [1, 7] this is reminiscent of the obstruction to the generation of axial (and topological) symmetries in 3d $\mathcal{N} = 2$ gauge theories. Here in the 2d picture we do not have an analogous candidate playing the role of the monopole superpotential[3], nevertheless we claim that the identity derived from the reduction of the zero flux sector of the $S^2 \times T^2$ topologically twisted index to the elliptic genus is reliable, and can be used to provide a derivation of the dualities under investigation. Assuming the matching of the elliptic genera for such duality, we will provide the relative identity for the duality involving SU(N) with two antisymmetric chirals and four fundamentals. Such proof requires a recursive analysis that leads to the towers of operators, expected from the field theory duality, that one can conjecture from the 3d analogous found in [27]. Once this duality with $n_f = 4$ is established, we will show that all the other cases with $n_a \neq 0$ can be derived from the one with $n_f = 4$, *i.e.* in the recursive derivation we always find a deformed version of the model with $n_f = 4$.

In the second part of the paper we revisit the problem of 2d dualities without a known 4d parent, by considering the case of SU($2n$) with an antisymmetric flavor and four fundamentals discussed in [9]. Here we present a 4d parent duality, with a non-vanishing superpotential, that reduces to the 2d expected one by twisting along an assignation of non-negative integer R charges.

The last part of the paper focuses on SU(N) and USp($2n$) dualities in presence of a charged antisymmetric Fermi field. These models are obtained by reducing 4d dualities that have been guessed in the literature from the analysis of the superconformal index [30]. A physical derivation of such 4d parent dualities was then provided in [20, 21]. We study the twist of these dualities on the two sphere by choosing integer non-vanishing R symmetries and in this way we argue the existence of new 2d dualities. In the first case the duality involves an SU(N) theory with N fundamentals and one antisymmetric Fermi whose dual model is identified with an LG model. The second case regards a duality between an USp($2n$) theory and an USp($2N-2$) theory, both with an antisymmetric Fermi, $4N$ fundamentals and non-trivial J-terms.

2 Basic dualities

In this section we review some of the relevant basic dualities that we will use in the paper in order to derive the matching of the elliptic general for the dualities proposed below. These dualities are already known, but they have been less discussed in the

[3]The notion of boundary monopoles in [29] seems promising for understanding a dynamical origin for such obstructions.

literature, for this reason we provide a detailed discussion and some further comments in order to employ them below.

In general we consider the 4d/2d reduction of dualities by compactifying on S^2 and by applying the prescription introduced in [1]: *i.e.* given a 4d chiral multiplet with R charge R, this reduces to

- $R-1$ Fermi multiplets if $R>1$,
- $1-R$ chiral multiplets if $R<1$,
- no multiplets if $R=1$,

while the 4d vector multiplet reduces to a 2d vector multiplet. As discussed in [1] this set of rules, provided that we have integer and non-negative R charges, allows to obtain a 2d $\mathcal{N}=(0,2)$ theory from a 4d $\mathcal{N}=1$ one. Here we further restrict our attention to cases without any field with $R>2$ in order to avoid the generation of possible non-abelian symmetries in 2d.

2.1 USp($2n$) with $2n+3$ $\square$ chirals and 1 $\square$ Fermi

We start our survey by discussing a duality with a 4d origin that will play a relevant role in our search of new 2d gauge/LG dualities. It corresponds to USp($2n$) with $2n+3$ fundamental chirals and one Fermi. The duality can be derived from 4d using USp($2n$) with $2n+4$ fundamentals by assigning an R charge equal to 2 to one fundamental and R charges equal to zero to the remaining $2n+3$ fundamentals. The 2d field content is anomaly free and the dual meson splits into a chiral antisymmetric meson with $(2n+3)(n+2)$ components $\Phi_{M_{ab}}$ and $2n+3$ Fermi Ψ_a.

The superpotential of the 2d theory is

$$W_{2d}=\epsilon_{a_1,\ldots,a_{2n+3}}\left(\Phi_{M_{a_1a_2}}\ldots\Phi_{M_{a_{2n+1}a_{2n+2}}}\Psi_{a_{2n+3}}\right)\equiv\Phi_M^{n+1}\Psi \tag{2.1}$$

and it descends from the 4d superpotential of the confining theory, corresponding to the limiting case of the Intriligator-Pouliot duality [6]. In the following, we will adopt a 2d notation, where we will refer to the J-term for the Fermi Ψ_a instead of the 2d superpotential. Moreover, in all the examples discussed in this paper the E-terms are vanishing.

The global charges of the fields are

$$\begin{array}{c|cc} & \mathrm{U}(1)_Q & \mathrm{SU}(2n+3)\\ \hline Q & 1 & \square\\ \psi & -2n-3 & \cdot\\ \hline \Phi_M=Q^2 & 2 & \begin{array}{c}\square\\[-4pt]\square\end{array}\\ \Psi=Q\psi & -2n-2 & \square \end{array} \tag{2.2}$$

Figure 1: In this figure we represents the quivers associated to the duality between USp($2n$) with $2n + 3$ fundamental chirals and one Fermi, and the LG model. Dashed lines are for Fermi fields while continuous lines are for chiral multiplets. The dots refer to the singlets.

As a first check one can see that the anomalies

$$\kappa_{QQ} = -4n(n+1)(2n+3)\,, \qquad \kappa_{\mathrm{SU}(2n+3)^2} = n\,, \tag{2.3}$$

are equal on both sides of the duality.

Here is a crucial aspect of the duality proposed above: on the gauge theory side of the duality we have not included an axial symmetry, which gives opposite charge to the $2n + 3$ chirals and to the Fermi. However, such symmetry is not forbidden by the field content and by the requirements from anomaly cancellation. At the level of localization such a symmetry is not generated in the reduction of the $S^2 \times T^2$ index to the 2d elliptic genus along the lines of the prescription of [1], i.e. by assigning vanishing R charge to $2n+2$ fields and $R = 2$ to the $(2n+4)$-th 4d $\mathcal{N} = 1$ chiral. As discussed originally in [1] and elaborated in [7], in this case the absence of such a symmetry is very similar to the obstruction to the generation of an axial symmetry in the 4d/3d reduction of dualities due to finite size effects [31]. In the 4d/2d reduction of [1], restricting our attention to the cases with $R = 0, 1$ and 2, such analogy is ubiquitous for all the anomaly free R charge assignations where there are no fields with $R = 1$, *i.e.* each 4d field gives origin either to a 2d $\mathcal{N} = (0, 2)$ chiral or to a 2d $\mathcal{N} = (0, 2)$ Fermi.

We expect that analogous finite size effects have to be considered in such 2d cases as well. Even if we postpone a physical discussion about the origin of such effect to future investigation, it is worth here to comment more on such issue. At the level of localization the obstruction imposed by the anomalies translates into a constraint on the fugacities. Borrowing the mathematical terminology, such constraints are denoted as balancing conditions in the literature of the 4d superconformal index (see for example [30]). Similar constraints arise in the analysis of the 3d three sphere partition function when monopole superpotential (for example KK monopole superpotentials signalling the presence of finite size effects) are considered. Similarly, in the 4d/2d reduction of the $S^2 \times T^2$ index to the elliptic genus, the 4d constraints on the fugacities survive when only fields with R charges $R = 0$ and $R = 2$ are considered in 4d. For this reason we expect that such balancing conditions signal the fact that finite size effects have to be considered in such cases. We will return on such issue below in the paper when we will apply the duality discussed here in order to deconfine the antisymmetric tensors[4].

At the level of the elliptic genus the duality discussed in this subsection translates into the identity

$$I^{(2n+3;\,\cdot)}_{\mathrm{USp}(2n)}(x\vec{u};x^{-2n-3};\cdot) = \frac{\prod_{a=1}^{2n+3}\theta\left(qu_a/x^{2n+2}\right)}{\prod_{1\leq a<b\leq 2n+3}\theta(x^2u_au_b)}\,, \tag{2.4}$$

with $\prod_{a=1}^{2n+3}u_a = 1$. We refer the reader to Appendix A for the conventions adopted here on the elliptic genus.

2.2 USp($2n$) with $2n+2$ fundamentals

This duality has been obtained in [1] and further studied in [32] for $n = 1$. It can be obtained from the reduction of the 4d s-confining limit of the USp($2n$) Intriligator-Pouliot duality, *i.e.* in presence of $2n+4$ fundamentals and vanishing superpotential. In this case the twist needs to be performed by fixing the R charge of two fundamentals to $R = 1$, and by assigning vanishing R charge to the remaining ones.

The 2d duality maps an USp($2n$) gauge theory with $2n+2$ fundamental chirals Q to an LG theory with an antisymmetric chiral $A = Q^2$ in addition to a Fermi Ψ with J-term $J_\Psi = \mathrm{Pf}A$. At the level of the elliptic genus the relative identity is

$$I^{(2n+2,\cdot)}_{\mathrm{USp}(2n)}(\vec{u};\cdot;\cdot) = \frac{\theta\left(q(u_1\dots u_{2n+2})^{-1}\right)}{\prod_{1\leq a<b\leq 2n+2}\theta(u_au_b)}\,, \tag{2.5}$$

where we stress that the numerator on the RHS is written in this way in order to make apparent the structure of the J-term for the Fermi Ψ. Using the fact that $\theta(q/x) = \theta(x)$ we could also write the numerator as $\theta(\prod u_a)$, which would correspond to exchanging the J term with an E term. We refer the reader to Appendix A for the conventions adopted here on the elliptic genus.

2.3 SU(N) with $N+x$ $\square$ chirals, $N-x+y$ $\overline{\square}$ chirals, y $\overline{\square}$ Fermi

Here we present the duality introduced in [1]. We first start from a 4d SU(N) gauge theory with $N+x$ flavors. The Seiberg dual of this theory is a 4d SU(x) gauge theory with $N+x$ flavors, a chiral meson and a superpotential given by $W = \tilde{q}\varphi q$. An anomaly-free R symmetry for the electric theory requires that

$$\sum_{i=1}^{N+x}\frac{1}{2}\big(R(Q_i)-1\big) + \sum_{i=1}^{N+x}\frac{1}{2}\big(R(\tilde{Q}_i)-1\big) + N = 0\,. \tag{2.6}$$

[4]Observe that this 2d confining duality has been recently used in [8] in order to prove 2d confining dualities with antisymmetric matter.

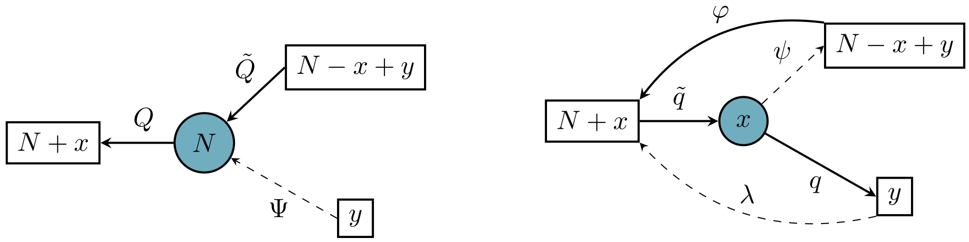

Figure 2: Quiver diagrams of the electric theory (left) and the magnetic theory (right).

Any R charge assignation must respect this constraint. Using the 4d duality dictionary, in the magnetic theory we have

$$R(q_i) = \frac{1}{x}\sum_{j=1}^{N+x} R(\tilde{Q}_j) - R(\tilde{Q}_i)\,, \qquad R(\tilde{q}_i) = \frac{1}{x}\sum_{j=1}^{N+x} R(Q_j) - R(Q_i)\,, \\ R(\varphi_{ij}) = R(Q_i) + R(\tilde{Q}_j)\,. \tag{2.7}$$

To perform the reduction to 2d we assign $R=0$ to $N+x$ fundamental chirals and to $N-x+y$ antifundamental chirals, $R=1$ to some other $2(x-y)$ antifundamental chirals and $R=2$ to the last y antifundamental chirals. One can easily check that such assignation respects the constraint (2.6). Therefore we obtain a 2d theory with $N+x$ fundamental chiral multiplets Q, $N-x+y$ antifundamental chiral multiplets $\tilde{Q}$ and y antifundamental Fermi multiplets Ψ, as shown on the left side of the Figure 2. There is a restriction on the allowed values of x and y, indeed Seiberg duality requires $x \geq 2$ and from the reduction rules we see that $x > y$ [5].

On the other hand, from (2.7) the field content of the 4d magnetic theory reduces as follows: the 4d antifundamental chirals become $N+x$ antifundamental chirals $\tilde{q}$ in 2d, the 4d fundamental chirals split into y fundamental chirals q and $N-x+y$ fundamental Fermi fields[6] ψ in 2d. The 4d chiral meson splits into a 2d chiral meson φ and a 2d Fermi meson λ. The magnetic theory is represented on the right side of Figure 2. The original 4d superpotential splits into two J-terms

$$J_\lambda = \tilde{q}q, \qquad J_\psi = \tilde{q}\varphi\,. \tag{2.8}$$

In order to check this new 2d duality we verify that the 't Hooft anomalies coincide. The global symmetries of both electric and magnetic theories are listed below.

[5]The case $x=y$ is similar to the one studied in subsection 2.1, where a 4d anomalous axial symmetry is not generated in 2d. We will not need such duality in the present discussion and for this reason we omit to review this case.

[6]Or equivalently antifundamentals, trading the relative J terms with opportune E-terms.

	$\mathrm{SU}(N+x)$	$\mathrm{SU}(N-x+y)$	$\mathrm{SU}(y)$	$\mathrm{U}(1)_Q$	$\mathrm{U}(1)_{\tilde{Q}}$	$\mathrm{U}(1)_\Psi$	$\mathrm{U}(1)_R$
Q	$\overline{\square}$	$\cdot$	$\cdot$	1	0	0	0
$\tilde{Q}$	$\cdot$	$\square$	$\cdot$	0	1	0	0
Ψ	$\cdot$	$\cdot$	$\square$	0	0	-1	1
q	$\cdot$	$\cdot$	$\overline{\square}$	$-1-\frac{N}{x}$	0	1	0
$\tilde{q}$	$\square$	$\cdot$	$\cdot$	$\frac{N}{x}$	0	0	0
ψ	$\cdot$	$\overline{\square}$	$\cdot$	$-1-\frac{N}{x}$	-1	0	1
φ	$\overline{\square}$	$\square$	$\cdot$	1	1	0	0
λ	$\overline{\square}$	$\cdot$	$\square$	1	0	-1	1

(2.9)

Notice that we have an explicit duality dictionary to write $\tilde{q}$, φ and λ in terms of the electric variables, but not for q and ψ. However, we can fix their $\mathrm{U}(1)$ charges from the superpotential (2.8). From the table above one can read the 't Hooft anomalies and find out that they match:

$$\begin{aligned}
&\kappa_{\mathrm{SU}(n)^2} = \frac{N}{2}\,, && \kappa_{QQ} = -\kappa_{QR} = N(N+x)\,,\\
&\kappa_{RR} = N^2+1\,, && \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R} = N(N-x+y)\,,\\
&\kappa_{Q\tilde{Q}} = \kappa_{Q\Psi} = \kappa_{\tilde{Q}\Psi} = 0\,, && \kappa_{\Psi\Psi} = -\kappa_{\Psi R} = -Ny\,.
\end{aligned} \tag{2.10}$$

At the level of the elliptic genus the identity that follows from the reduction of the index on $S^2\times T^2$ is

$$I^{(N+x;N-x+y;y;\cdot;\cdot)}_{\mathrm{SU}(N)}(\vec{u};\vec{v};\vec{h};\cdot;\cdot) = \frac{\prod_{i=1}^{N+x}\prod_{a=1}^{y}\theta(qu_ih_a)}{\prod_{i=1}^{N+x}\prod_{j=1}^{N-x+y}\theta(u_iv_j)} I^{(y;N+x;N-x+y;\cdot;\cdot)}_{\mathrm{SU}(x)}(\vec{\tilde{h}},\vec{\tilde{u}},\vec{\tilde{v}};\cdot;\cdot) \tag{2.11}$$

with $\tilde{u}_i = u_i^{-1}\prod_{\ell=1}^{N+x}u_\ell^{\frac{1}{x}}$, $\tilde{v}_j = v_j^{-1}\prod_{\ell=1}^{N+x}u_\ell^{-\frac{1}{x}}$ and $\tilde{h}_a = h_a^{-1}\prod_{\ell=1}^{N+x}u_\ell^{-\frac{1}{x}}$. Again, we refer the reader to Appendix A for the conventions adopted here on the elliptic genus.

3 SU(N) with two antisymmetric chirals

In this section we discuss the existence of LG duals for 2d $\mathcal{N}=(0,2)$ $\mathrm{SU}(N)$ gauge theories with two antisymmetric tensors $A_{1,2}$, n_f fundamentals and n_a antifundamentals, with $n_f+n_a=4$. The cases with even $N=2n$ rank and with odd $N=2n+1$ rank are different for any allowed choice of n_f and n_a, therefore we need to treat each case separately. In each one we propose the duality and check the 't Hooft anomaly matching. Furthermore, we provide a derivation of the matching of the elliptic genera by employing other identities obtained by the reduction of 4d Seiberg and Intriligator-Pouliot duality.

3.1 SU($2n$) with four fundamentals

In this case the dual LG model has three gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

	SU($2n$)	SU(2)	SU(4)	U(1)$_A$	U(1)$_Q$	U(1)$_{R_0}$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	$\cdot$	1	0	0
Q	$\square$	$\cdot$	$\square$	0	1	0
T_n	$\cdot$	$\otimes^n_{\text{sym}}\square$	$\cdot$	n	0	0
T_{n-1}	$\cdot$	$\otimes^{n-1}_{\text{sym}}\square$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$n-1$	2	0
T_{n-2}	$\cdot$	$\otimes^{n-2}_{\text{sym}}\square$	$\cdot$	$n-2$	4	0
Ψ	$\cdot$	$\otimes^{2n-2}_{\text{sym}}\square$	$\cdot$	$-2n+2$	-4	1

(3.1)

The three chirals T_j correspond to the following gauge invariant combinations of the charged fields

$$T_n = A^n, \quad T_{n-1} = A^{n-1}Q^2, \quad T_{n-2} = A^{n-2}Q^4\,, \tag{3.2}$$

while the J-term is

$$J_\Psi = T_n T_{n-2} + T_{n-1}^2\,, \tag{3.3}$$

We have computed the 't Hooft anomalies in the two phases and we have observed that they match. Explicitly, we have

$$\begin{aligned}
&\kappa_{\mathrm{SU}(2)^2} = \tfrac{n(2n-1)}{2}\,, && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n-1)\,,\\
&\kappa_{\mathrm{SU}(4)^2} = n\,, && \kappa_{QQ} = -\kappa_{QR_0} = 8n\,,\\
&\kappa_{R_0R_0} = 6n+1\,, && \kappa_{AQ} = 0\,.
\end{aligned} \tag{3.4}$$

The further evidence that we provide to corroborate the existence of this duality consists of showing that the matching of the elliptic genera of the two phases is a consequence of other identities. These can be derived from the matching of the $S^2\times T^2$ topologically twisted index of well stated 4d s-confining parent dualities for special unitary and symplectic SQCD.

The identity that we want to prove is

$$\begin{aligned}
&I^{(4,\cdot,\cdot,2,\cdot)}_{\mathrm{SU}(2n)}(\vec{u};\cdot;\cdot;\vec{t};\cdot) = \\
&= \frac{\prod_{j=0}^{2n-2}\theta\left(q/\left(t_1^{2(2n-2-j)}t_2^{2j}\prod_{a=1}^4 u_a\right)\right)}{\prod_{j=0}^{n}\theta\big(t_1^{2(n-j)}t_2^{2j}\big)\prod_{a<b}^4\prod_{j=0}^{n-1}\theta\big(t_1^{2(n-1-j)}t_2^{2j}u_au_b\big)\prod_{j=0}^{n-2}\theta\big(t_1^{2(n-2-j)}t_2^{2j}\prod_{a=1}^4 u_a\big)}\,,
\end{aligned} \tag{3.5}$$

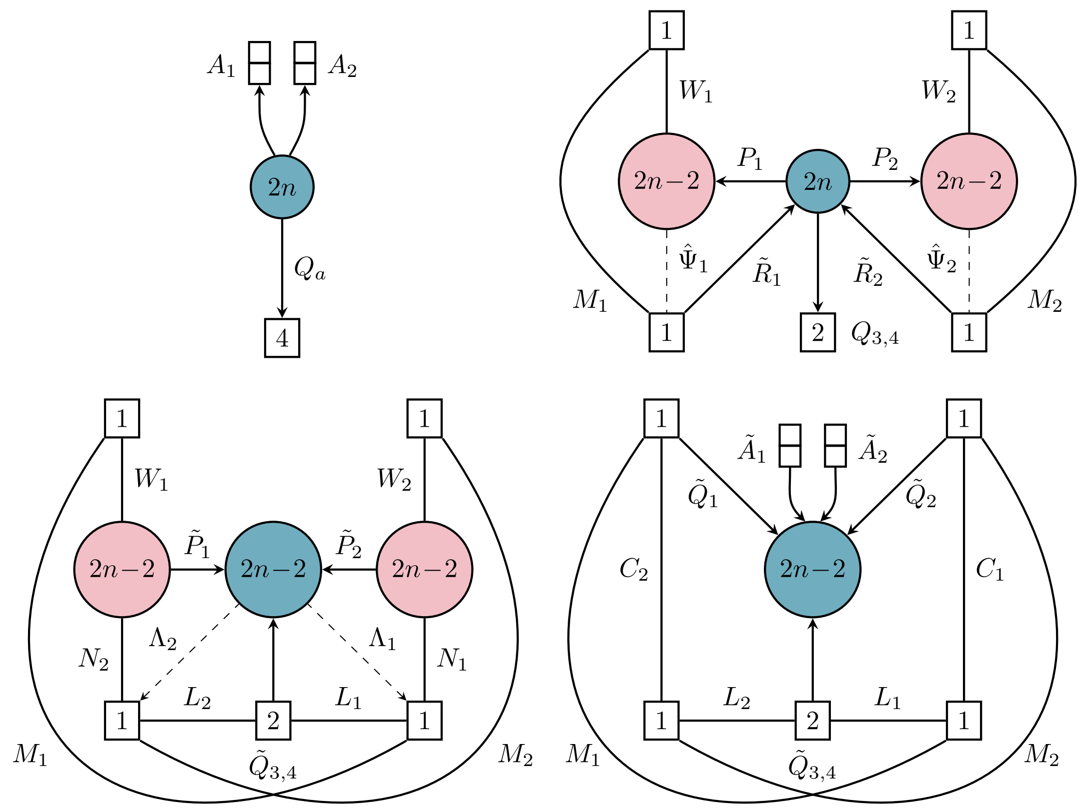

Figure 3: Deconfinement steps of SU($2n$) with two antisymmetrics and four fundamentals.

where we refer to the appendix A for the details on the conventions on the elliptic genus used here. The collection of θ-functions on the RHS of the identity (3.5) represents the Fermi Ψ (numerator) and the chirals T_j (denominator) in the dual LG theory.

In the following, in order to prove the identity, we will break some of the global symmetries by moving all the theta functions from the denominator of the RHS to the numerator of LHS of (3.5). We will further employ the relation

$$\theta(x) = \theta(q/x)\,. \tag{3.6}$$

At the field theory level this operation corresponds to adding a J-term on the gauge theory side by setting a gauge invariant combination of chiral fields (the one that we have moved from the RHS to the LHS) to zero using the equations of motion. The net effect of this operation on the elliptic genus consists of adding a Fermi on the gauge theory side and of removing a chiral on the LG side. We denote the new Fermi that appears on the LHS as Fermi *flipper*, borrowing the terminology used in higher dimensional theories with four supercharges for chiral multiplets.

In this case we also redefine the singlets T_n, T_{n-1} and T_{n-2} as $\mathbf{T}_{n-j,j}$, for $j = 0,\ldots,n-1$, $\mathbf{T}^{ab}_{n-j-1,j}$ for $j = 0,\ldots,n$ and $1 \leq a < b \leq 4$ and $\mathbf{T}_{n-j-2,j}$ for $j = 0,\ldots,n-2$ respectively. This redefinition is quite useful in the following, because we will explicit break the SU(2) flavor symmetry and expressing the fields as $\mathbf{T}$ allows us to identify the $\mathrm{U}(1) \subset \mathrm{SU}(2)$ charges. Furthermore, the Fermi Ψ is redefined as $\mathbf{\Psi}_{2n-2-j,j}$ with $j = 0,\ldots,2n-2$ accordingly. The electric theory is then modified by flippling all the combinations $\mathbf{T}$, which corresponds to adding the following J-terms

$$\begin{aligned}
&\Psi_{\mathbf{T}_{n-j,j}} = A_1^{n-j}A_2^j\,, && j = 0,\ldots,n\,,\\
&\Psi_{\mathbf{T}^{ab}_{n-j-1,j}} = A_1^{n-j-1}A_2^jQ_aQ_b\,, && j = 0,\ldots,n-1,\ 1 \leq a < b \leq 4\,,\\
&\Psi_{\mathbf{T}_{n-j-2,j}} = A_1^{n-j-2}A_2^jQ_1Q_2Q_3Q_4\,, && j = 0,\ldots,n-2\,.
\end{aligned} \tag{3.7}$$

In the dual theory, we are then left with only the $2n-1$ free Fermi $\mathbf{\Psi}_{2n-2-j,j}$.

Then the crucial aspect of the analysis consists of finding an auxiliary dual quiver gauge theory with new gauge groups and new bifundamentals, where the original antisymmetric fields $A_{1,2}$ are absent. In the case at hand, such an auxiliary quiver corresponds to the second one in Figure 3.

There are two symplectic gauge groups $\mathrm{USp}(2n-2)_{1,2}$ and each one is characterized by one USp/SU bifundamental $P_{1,2}$, one fundamental $W_{1,2}$ and one Fermi $\hat{\Psi}_{1,2}$. There are also two new $\mathrm{SU}(2n)$ fundamentals $\hat{R}_{1,2}$ and two J-terms

$$J_{\hat{\Psi}_{1,2}} = \tilde{R}_{1,2}P_{1,2} + W_{1,2}M_{1,2}\,. \tag{3.8}$$

Observe that the procedure manifestly breaks the $\mathrm{SU}(4)\times\mathrm{SU}(2)$ global symmetry. The original fields $A_{1,2}$ and $Q_{1,2}$ correspond here to the combinations $P_{1,2}^2$ and $P_{1,2}W_{1,2}$ respectively. In addition, the J-terms (3.7) are modified as

$$\begin{aligned}
&\Psi_{\mathbf{T}_{n-j,j}} = P_1^{2n-2j}P_2^{2j}\,, && j = 0,\ldots,n\,,\\
&\Psi_{\mathbf{T}^{1,2}_{n-j-1,j}} = P_1^{2(n-j-1)}P_2^{2j}P_1W_1P_2W_2\,, && j = 0,\ldots,n-1\,,\\
&\Psi_{\mathbf{T}^{\ell,a}_{n-j-1,j}} = P_1^{2(n-j-1)}P_2^{2j}P_\ell W_\ell Q_a\,, && j = 0,\ldots,n-1,\ \ell = 1,2,\ a = 3,4\,,\\
&\Psi_{\mathbf{T}^{3,4}_{n-j-1,j}} = P_1^{2(n-j-1)}P_2^{2j}Q_3Q_4\,, && j = 0,\ldots,n-1\,,\\
&\Psi_{\mathbf{T}_{n-j-2,j}} = P_1^{2(n-j-2)}P_2^{2j}P_1W_1P_2W_2Q_3Q_4\,, && j = 0,\ldots,n-2\,.
\end{aligned} \tag{3.9}$$

The original quiver in Figure 3 can be recovered if we use the duality reviewed in subsection 2.1, provided we remove an extra non-anomalous axial symmetry that is allowed by the field content.

As we stressed above, we do not have a dynamical mechanism that removes such a symmetry. A similar problem arises in 3d, where the extra symmetry is removed by the presence of a (linear) monopole superpotential. The procedure carried out here was indeed applied in [22] to the analysis of a 3d duality with the same electric matter content[7] found in [27], where in the auxiliary quiver two linear monopole superpotentials terms were added to the theory in order to remove the axial symmetries. This analogy is helpful in order to justify the existence of a dynamical constraint also in 2d, because the models discussed here can be obtained from the 3d ones as boundary dualities along the lines of the construction in [29]. If we apply such construction to the auxiliary quiver we then have to consider the contribution of the boundary monopoles, that forbids the generation of the axial symmetry in the half index. Here a similar constraint exists also at the level of the elliptic genus. This last observation descends from the reduction of the $S^2 \times T^2$ partition function to the elliptic genus, where the choice of R charges discussed in subsection 2.1, prevents the extra symmetry to be generated.

Having this caveat in mind we can proceed by dualizing the $\mathrm{SU}(2n)$ gauge node. This model has $4n-2$ fundamentals and 2 antifundamentals and the necessary duality has been reviewed in subsection 2.3. Here we consider the case with $y = 0$.

The model obtained in this case is represented in the third quiver of figure 3. The non-trivial aspect of this model is that the constraints that we imposed on the axial symmetries for the $\mathrm{USp}(2n-2)$ nodes above are not required here anymore. The reason is that there are linear J-term in the dual phase that remove some of the Fermi and chiral fields, and non-anomalous charges in the leftover field content are exactly the ones visible in the matching of the elliptic genera.

In this case the J-terms for the Fermi $\Lambda_{1,2}$ obtained in the duality are

$$\begin{aligned} J_{\Lambda_1} &= \sum_{a=3,4} \tilde{Q}_a L_{1,a} + \tilde{P}_1 M_1 W_1 + N_1 \tilde{P}_2 \,, \\ J_{\Lambda_2} &= \sum_{a=3,4} \tilde{Q}_a L_{2,a} + \tilde{P}_2 M_2 W_2 + N_2 \tilde{P}_1 \,. \end{aligned} \tag{3.10}$$

The last step in Figure 3 corresponds to confining the two $\mathrm{USp}(2n-2)$ gauge nodes. In this case, they both have $2n$ fundamentals and the dual models are LG theories, where the two combinations $\tilde{P}_{1,2}^2$ are conjugated antisymmetric chirals. There are also four $\mathrm{SU}(2n-2)$ antifundamentals, and the J-terms obtained after solving the equation of motion are

$$J_{\Psi_1^{(0)}} = \tilde{Q}_1 \tilde{A}_1^{n-2} \Big(\sum_{a=3,4} \tilde{Q}_a L_{2,a} + M_2 \tilde{Q}_2 \Big) + C_1 \mathrm{Pf} \tilde{A}_1 \,, \tag{3.11}$$

$$J_{\Psi_2^{(0)}} = \tilde{Q}_2 \tilde{A}_2^{n-2} \Big(\sum_{a=3,4} \tilde{Q}_a L_{1,a} + M_1 \tilde{Q}_1 \Big) + C_2 \mathrm{Pf} \tilde{A}_2 \,. \tag{3.12}$$

[7] More precisely the duality that was proved in [22] through tensor deconfinement had 4 fundamental and two antisymmetric chirals 3d $\mathcal{N} = 2$.

Notice that we have denoted the Fermi flippers, arising from the gauge/LG duality of the symplectic gauge nodes discussed in subsection 2.2, as $\Psi_1^{(0)}$ and $\Psi_2^{(0)}$, as they will constitute the *extremal* fields in the Fermi *tower* denoted as Ψ in equation (3.1).

In this way we have obtained a duality between the original SU($2n$) model with two antisymmetric and four fundamental chirals, and an SU($2n-2$) model with two conjugate antisymmetric and four antifundamental chirals, in addition to a set of Fermi and chiral singlets.

We now iterate the procedure until we reach an SU($2n-4$) gauge theory with the same charged field content of the original one, *i.e.* two antisymmetric and four fundamental chirals. In this way we have a simpler relation between the elliptic genera, that can be recursively applied to obtain the final relation between the original model and an LG model. Such relation is

$$\begin{aligned} I_{\mathrm{SU}(2n)}^{(4,\cdot,\cdot,2,\cdot)}(\vec{u};\cdot;\cdot;\vec{t};\cdot) &= I_{\mathrm{SU}(2n-4)}^{(4,\cdot,\cdot,2,\cdot)}\left((t_1t_2)^{\frac{1}{n-2}}\vec{u};\cdot;\cdot;(t_1t_2)^{\frac{1}{n-2}}\vec{t};\cdot\right) \\ &\times\frac{\theta\left(q/(t_{2,1}^{4n-4}\prod_{a=1}^4 u_a)\right)\,\theta\left(q/(t_{1,2}^2t_{2,1}^{4n-6}\prod_{a=1}^4 u_a)\right)}{\theta\left(t_{1,2}^{2n}\right)\,\theta\left(t_{1,2}^{2n-4}\prod_{a=1}^4 u_a\right)\,\prod_{a<b}\theta\left(t_{1,2}^{2n-2}u_au_b\right)}\,. \end{aligned} \tag{3.13}$$

The relation is derived by applying the various steps of deconfinement, duality and reconfinement discussed above and summarized graphically in Figure 3 twice. The relations used to derive (3.13) have been summarized in Section 2 and they correspond to (2.4), (2.11) and (2.5) respectively. Observe that we kept the chiral fields unflipped in the relation (3.13), while they appeared as Fermi fields in the field theory discussion in the SU($2n$) side.

Each term in the second line of the (3.13) corresponds to one of the expected singlets of the duality, in which the procedure has apparently broken the global SU(2) non-abelian global symmetry[8]. More concretely:

- $\theta(t_{1,2}^{2n})$ corresponds to the combinations $A_{1,2}^n$
- $\theta(t_{1,2}^{2n-4}\prod_{a=1}^4 u_a)$ corresponds to $A_{1,2}^{n-2}Q_1Q_2Q_3Q_4$
- $\prod_{a<b}\theta(t_{1,2}^{2n-2}u_au_b)$ corresponds to $A_{1,2}^{n-1}Q_aQ_b$, antisymmetric in the SU(4) indices
- $\theta\left(q/(t_{2,1}^{4n-4}\prod_{a=1}^4 u_a)\right)$ correspond to the two Fermi $\Psi_{2n-2,0}^{(0)}$ and $\Psi_{0,2n-2}^{(0)}$
- $\theta\left(q/(t_{1,2}^2t_{2,1}^{4n-6}\prod_{a=1}^4 u_a)\right)$ correspond to the two Fermi $\Psi_{2n-1,1}^{(0)}$ and $\Psi_{1,2n-1}^{(0)}$.

[8]Also the SU(4) global symmetry is broken by our procedure, but it is actually recovered in the SU($2n-4$) theory.

The (0) superscript in the Fermi fields indicates that they have been created at the *zero*-th iteration of the procedure.

A recursive application of this relation leads to an SU(4) gauge theory model for even n or to an SU(2) gauge theory for odd n. In general, at the j-th step we obtain an SU$(2n-4j)$ gauge theory, and the leftover integral is

$$I^{(4,\cdot,\cdot,2,\cdot)}_{\mathrm{SU}(2n-4j)}\left((t_1t_2)^{\frac{j}{n-2j}}\vec{u};\cdot;\cdot;(t_1t_2)^{\frac{j}{n-2j}}\vec{t};\cdot\right). \tag{3.14}$$

The Fermi and chiral singlets generated at this step contribute as

- $\theta(t_{1,2}^{2(n-j+1)}t_{2,1}^{2(j-1)})$ corresponding to the combinations $A_{1,2}^{n-j+1}A_{2,1}^{j-1}$
- $\prod_{a<b}\theta(t_{1,2}^{2(n-j)}t_{2,1}^{2(j-1)}u_au_b)$ corresponding to $A_{1,2}^{n-j}A_{2,1}^{j-1}Q_aQ_b$, antisymmetric in the SU(4) indices
- $\theta(t_{1,2}^{2(n-j-1)}t_{2,1}^{2(j-1)}\prod_{a=1}^4 u_a)$ corresponding to $A_{1,2}^{n-j-1}A_{2,1}^{j-1}Q_1Q_2Q_3Q_4$
- $\theta(q/(t_{1,2}^{4(n-j)}t_{2,1}^{4(j-1)}\prod_{a=1}^4 u_a))$ associated to the two Fermi $\Psi^{(j)}_{2n-2j,2j-2}$ and $\Psi^{(j)}_{2j-2,2n-2j}$
- $\theta(q/(t_{1,2}^{2(2j-1)}t_{2,1}^{2(2n-2j-1)}\prod_{a=1}^4 u_a))$ corresponding to the two Fermi $\Psi^{(j)}_{2n-2j-1,2j-1}$ and $\Psi^{(j)}_{2j-1,2n-2j-1}$.

The Fermi generated at this stage constitute another bit of the tower of states Ψ.

In order to complete the proof of (3.5) we must distinguish the cases $n=2m$ and $n=2m+1$ as discussed above.

The case of SU$(4m)$

In this case we can use the recurrence relations until we get to an SU(4) gauge theory with two antisymmetric $\mathbf{A}_{1,2}$ and four fundamentals $\mathbf{Q}_{1,2,3,4}$. Their fugacities are $(t_1t_2)^{\frac{m-1}{2}}t_{1,2}$ and $(t_1t_2)^{\frac{m-1}{2}}u_a$ respectively.

This model is dual (see [9]) to an LG theory with chirals φ_i that contribute to the elliptic genus as

- $\varphi_1=\mathbf{A}_2\to\prod_{a<b}\theta\left(t_1^{2m-2}t_2^{2m}u_au_b\right)$
- $\varphi_2=\mathrm{Pf}\mathbf{A}_1\to\theta\left(t_1^{2m+2}t_2^{2m-2}\right)$
- $\varphi_3=\mathbf{A}_1\mathbf{Q}^2\to\prod_{a<b}\theta\left(t_1^{2m}t_2^{2m-2}u_au_b\right)$
- $\varphi_4=\mathbf{Q}^4\to\theta\left((t_1t_2)^{2(m-1)}\prod_{a=1}^4 u_a\right)$
- $\varphi_5=\mathrm{Pf}\mathbf{A}_2\to\theta\left(t_1^{2m-2}t_2^{2m+2}\right)$

- $\varphi_6 = \mathbf{A}_1\mathbf{A}_2 \to \theta\left((t_1t_2)^{2m}\right)$.

There are in addition three Fermi singlets, whose charges (and contribution to the elliptic genus) can be read from the J-terms[9]. We have

- $J_{\Psi_{11}} \supset \varphi_1^2 \to \theta(q/(t_1^{4m}t_2^{4(m-1)}\prod_{a=1}^4 u_a))$
- $J_{\Psi_{33}} \supset \varphi_3^2 \to \theta(q/(t_1^{4(m-1)}t_2^{4m}\prod_{a=1}^4 u_a))$
- $J_{\Psi_{13}} \supset \varphi_1\varphi_3 \to \theta(q/(t_1^{2(2m-1)}t_2^{2(2m-1)}\prod_{a=1}^4 u_a))$.

These last Fermi singlets are the remaining ones needed for the full Fermi tower Ψ.

Once the contributions arising from the last duality are added to the ones obtained from the recursive relation above, we restore the original SU(2) symmetry and obtain the relation (3.5) for $n = 2m$.

The case of $\mathrm{SU}(4m+2)$

In order to complete the proof of (3.5) we need to discuss the case $\mathrm{SU}(4m+2)$ as well. In this case we arrive at an SU(2) gauge theory with four fundamentals $\mathbf{Q}_{1,2,3,4}$ with fugacities $(t_1t_2)^{2m}u_a$ after m iterations. The two antisymmetric are singlets and they contribute to the elliptic genus as $\theta\big(t_{1,2}^{2(m+1)}t_{2,1}^{2(m-1)}\big)$. This gauge theory is dual to an LG model, where the singlets are given by the SU(4) two-index antisymmetric chiral $\mathbf{Q}_a\mathbf{Q}_b$ and a Fermi. The contribution of the chiral to the elliptic genus is $\prod_{a<b}\theta\big((t_1t_2)^{2m}u_au_b\big)$, while the Fermi contributes as $\theta\big(q/((t_1t_2)^{4m}\prod_{a=1}^4 u_a)\big)$, compatibly with the J-terms that would be generated in the duality in absence of flippers in the original $\mathrm{SU}(2n)$ model. Again, once the contributions of the last duality are added to the ones obtained from the recursive relation above, we restore the original SU(2) symmetry and obtain the relation (3.5) for $n = 2m+1$ as well.

This concludes the proof of the identity (3.5). Observe that at the level of the interaction we end up with a model with only Fermi multiplets, as we have flipped all the chirals T_j in the original gauge theory. In the following, we will actually use the un-flipped (or partially flipped) duality in order to prove that the other cases of $\mathrm{SU}(N)$ with two antisymmetric chirals, $n_f < 4$ fundamental chirals and $n_a > 1$ antifundamental chirals are dual to LG models.

[9]Here we have flipped all the chiral combinations $T_{n,n-1,n-2}$. Then, the LG model will only have free Fermi multiplets and vanishing J-terms. Nevertheless we can read their charges from the un-flipped duality discussed in [9].

3.2 $\mathrm{SU}(2n+1)$ with four fundamentals

In this case the dual LG model has two chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the following table

$$
\begin{array}{c|c|c|c|c|c|c}
 & \mathrm{SU}(2n+1) & \mathrm{SU}(2) & \mathrm{SU}(4) & \mathrm{U}(1)_A & \mathrm{U}(1)_Q & \mathrm{U}(1)_{R_0} \\
\hline
A & \begin{smallmatrix}\square\\[-2pt]\square\end{smallmatrix} & \square & \cdot & 1 & 0 & 0 \\
Q & \square & \cdot & \square & 0 & 1 & 0 \\
\hline
T_n & \cdot & \otimes^n_{\mathrm{sym}}\square & \square & n & 1 & 0 \\
T_{n-1} & \cdot & \otimes^{n-1}_{\mathrm{sym}}\square & \overline{\square} & n-1 & 3 & 0 \\
\Psi & \cdot & \otimes^{2n-1}_{\mathrm{sym}}\square & \cdot & -2n+1 & -4 & 2
\end{array}
\tag{3.15}
$$

The chirals T_j correspond to the following gauge invariant combinations of the charged fields

$$
T_n = A^n Q\,, \qquad T_{n-1} = A^{n-1}Q^3\,, \tag{3.16}
$$

while the J-term is

$$
J_\Psi = T_n T_{n-1}\,. \tag{3.17}
$$

We have computed the 't Hooft anomalies in the two phases and we have observed that they match. Explicitly we have

$$
\begin{aligned}
&\kappa_{\mathrm{SU}(2)^2} = \tfrac{n(2n+1)}{2}\,, \qquad && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n+1)\,,\\
&\kappa_{\mathrm{SU}(4)^2} = \tfrac{2n+1}{2}\,, && \kappa_{QQ} = -\kappa_{QR_0} = 8n+4\,,\\
&\kappa_{R_0R_0} = 6n+4\,, && \kappa_{AQ} = 0\,.
\end{aligned}
\tag{3.18}
$$

In the following we will provide a further check of this duality by showing that the elliptic genera of the two models match, provided the validity of other identities that already appeared in the literature, and that descend from the reduction of 4d dualities to 2d. The derivation is very similar to the one spelled out above, and for this reason we will be more sketchy and refer the reader to the derivation in subsection 3.1 for further details. The identity that we need to prove in order to check the validity of the duality is

$$
I^{(4,\cdot,\cdot,2,\cdot)}_{\mathrm{SU}(2n+1)} = \frac{\prod_{j=0}^{2n-1}\theta\left(q/\left(t_1^{2(2n-1-j)}t_2^{2j}\prod_{a=1}^4 u_a\right)\right)}{\prod_{a=1}^4\prod_{j=0}^{n}\theta\left(t_1^{2(n-j)}t_2^{2j}u_a\right)\prod_{a<b<c}^4\prod_{j=0}^{n-1}\theta\left(t_1^{2(n-1-j)}t_2^{2j}u_a u_b u_c\right)}\,. \tag{3.19}
$$

Again, we flip all the terms in the denominator of the RHS of (3.19) by moving the theta functions to the numerator of LHS and by using the relation (3.6). These theta functions are associated to Fermi flippers for the operators T_n and T_{n-1} on the gauge theory side. Then we re-define T_n and T_{n-1} as $\mathbf{T}^a_{n-j,j}$, for $j = 0,\ldots,n$ and $a = 1,\ldots,4$ and $\mathbf{T}^{abc}_{n-j-1,j}$ for $j = 0,\ldots,n-1$ and $1 \leq a < b < c \leq 4$ and the Fermi Ψ is redefined as $\mathbf{\Psi}_{2n-1-j,j}$ with $j = 0,\ldots,2n-1$ accordingly.

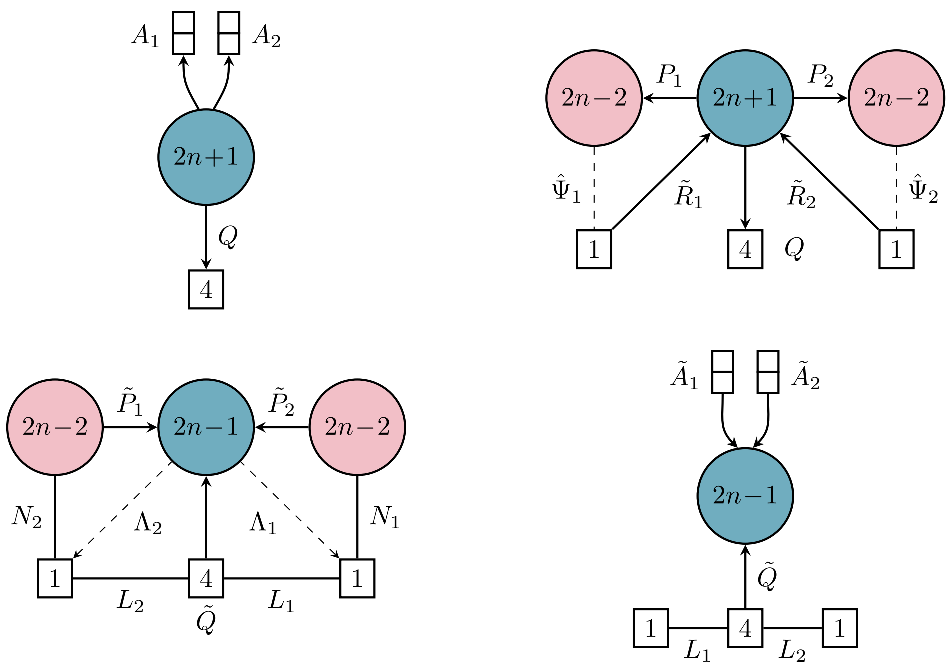

Figure 4: Deconfinement steps of SU$(2n+1)$ with two antisymmetrics and four fundamentals.

The electric theory is modified by the addition of the following J-terms

$$\begin{aligned} \Psi^a_{\mathbf{T}_{n-j,j}} &= A_1^{n-j} A_2^j Q_a\,, && j = 0,\ldots,n, \quad a = 1,\ldots,4\,, \\ \Psi_{\mathbf{T}^{abc}_{n-j-1,j}} &= A_1^{n-j-1} A_2^j Q_a Q_b Q_c\,, && j = 0,\ldots,n-1,\, 1 \leq a < b < c \leq 4\,. \end{aligned} \tag{3.20}$$

In the dual theory we are then left only with the free Fermi fields $\mathbf{\Psi}_{2n-1-j,j}$.

We proceed by finding an auxiliary quiver, shown in Figure 4, where the two antisymmetric chirals $A_{1,2}$ are traded with two USp$(2n-2)$ auxiliary gauge groups. In this case the process does not break the SU(4) global symmetry, differently from the case studied in subsection 3.1.

There are two USp/SU bifundamental $P_{1,2}$, two SU$(2n+1)$ fundamentals $\tilde{R}_{1,2}$ and two Fermi fields $\hat{\Psi}_{1,2}$. There are also two J-terms $J_{\hat{\Psi}_{1,2}} = \tilde{R}_{1,2} P_{1,2}$. The original fields $A_{1,2}$ correspond here to the combinations $P^2_{1,2}$. In addition, the J-terms (3.20) are modified as

$$\Psi_{\mathbf{T}^a_{n-j,j}} = P_1^{2n-2j} P_2^{2j} Q_a\,, \quad \Psi_{\mathbf{T}^{abc}_{n-j-1,j}} = P_1^{2(n-j-1)} P_2^{2j} Q_a Q_b Q_c\,. \tag{3.21}$$

The original quiver in Figure 4 can be recovered if we exploit the duality reviewed in subsection 2.1 provided that we remove an extra, non-anomalous, axial symmetry that is allowed by the field content.

This requirement is translated into a constraint imposed on the elliptic genus, which prevents fugacities for the axial symmetries to be turned on both for the chirals and the Fermi charged under the two $\mathrm{USp}(2n-2)$ factors. Again, we interpret the constraint as in the discussion of subsection (3.1).

We proceed by dualizing the $\mathrm{SU}(2n+1)$ gauge node. This model has $4n$ fundamentals and 2 antifundamentals and the duality we need has been reviewed in subsection 2.3. Here we consider the case with $y=0$. The model obtained in this case is represented in the third quiver of Figure 4. Again, the constraints imposed on the axial symmetries for the $\mathrm{USp}(2n-2)$ nodes above are not required here anymore. In this case the J-terms for the Fermi $\Lambda_{1,2}$ obtained in the duality are

$$J_{\Lambda_1} = L_1\tilde{Q} + N_1\tilde{P}_2\,, \qquad J_{\Lambda_2} = L_2\tilde{Q} + N_2\tilde{P}_1\,. \tag{3.22}$$

The last step in Figure 4 corresponds to confine the two $\mathrm{USp}(2n-2)$ gauge nodes. In this case they are both characterized by $2n$ fundamentals, and the dual models are LG theories, where the two combinations $\tilde{P}_{1,2}^2$ define conjugated antisymmetric chirals. There are also four $\mathrm{SU}(2n-2)$ antifundamentals, and the J-terms obtained after solving the equation of motion are

$$J_{\Psi_1^{(0)}} = \tilde{A}_1^{n-1}L_2\tilde{Q}\,, \qquad J_{\Psi_2^{(0)}} = \tilde{A}_2^{n-1}L_1\tilde{Q}\,. \tag{3.23}$$

Again we have denoted the Fermi flippers, arising from the gauge/LG duality of the symplectic gauge nodes discussed in subsection 2.2, as $\Psi_1^{(0)}$ and $\Psi_2^{(0)}$, as they will constitute the *extremal* fields in the Fermi *tower* denoted as Ψ in equation (3.15).

In this way we have obtained a duality between the original $\mathrm{SU}(2n+1)$ model with two antisymmetric and four fundamental chirals and an $\mathrm{SU}(2n-1)$ model with two conjugate antisymmetric and four antifundamental chirals, in addition to a set of Fermi and chiral singlets.

At this point of the discussion we iterate the procedure until we reach an $\mathrm{SU}(2n-3)$ gauge theory with the same charged field content of the original one, i.e. two antisymmetric and four fundamental chirals. In this way we get a simpler relation between the elliptic genera, that can be recursively applied in order to obtain the final relation between the original model and an LG model. Such relation is obtained by applying the relations (2.4), (2.11) and (2.5), and it is

$$I^{(4,\cdot,\cdot,2,\cdot)}_{\mathrm{SU}(2n+1)}(\vec{u};\cdot;\cdot;\vec{t};\cdot) \;= I^{(4,\cdot,\cdot,2,\cdot)}_{\mathrm{SU}(2n-3)}\left((t_1t_2)^{\frac{2}{2n-3}}\vec{u};\cdot;\cdot;(t_1t_2)^{\frac{2}{2n-3}}\vec{t};\cdot\right)$$

$$\times \frac{\theta\left(q/(t_{1,2}^{2(2n-1)}\prod_{a=1}^4 u_a)\right)\,\theta\left(q/(t_{1,2}^{2(2n-2)}t_{2,1}^2\prod_{a=1}^4 u_a)\right)}{\prod_{a=1}^4\theta\left(t_{1,2}^{2n}u_a\right)\prod_{a<b<c}\theta\left(t_{1,2}^{2n-2}u_a u_b u_c\right)}\,. \quad (3.24)$$

Analogously to the case studied in subsection 3.1 we keep the chiral fields unflipped at the level of the elliptic genus, while they appear as Fermi fields in the field theory discussion in the SU$(2n+1)$ side. Each term in the second line of the (3.24) corresponds to one of the expected singlets of the duality. These are consistent with the deconfinement procedure which apparently broke the global SU(2) non-abelian global symmetry. We leave to the interested reader the details of the matching in this case.

A recursive application of this relation leads to an SU(5) gauge theory model for even n or to an SU(3) gauge theory for odd n. In general, at the j-th step we obtain an SU$(2n+1-4j)$ gauge theory, and the leftover integral is

$$I^{(4,\cdot,\cdot,2,\cdot)}_{\mathrm{SU}(2n-4j+1)}\left((t_1t_2)^{\frac{2j}{2n+1-4j}}\vec{u};\cdot;\cdot;(t_1t_2)^{\frac{2j}{2n+1-4j}}\vec{t};\cdot\right)\,. \quad (3.25)$$

In addition, the Fermi and chiral singlets generated at this step can be worked out from the relation (3.24) in complete analogy to what was done in subsection 3.1. The details of the derivation are straightforward, thus we omit them.

To conclude the proof of (3.19), we must distinguish the cases $n=2m$ and $n=2m+1$.

The case of SU$(4m+1)$

In this case we can use the recurrence relations until we obtain an SU(5) gauge theory with two antisymmetric $\mathbf{A}_{1,2}$ and four fundamentals $\mathbf{Q}_{1,2,3,4}$. Their fugacities are $(t_1t_2)^{\frac{2(m-1)}{5}}t_{1,2}$ and $(t_1t_2)^{\frac{2(m-1)}{5}}u_a$ respectively.

Now, we can further deconfine the two antisymmetric tensors $\mathbf{A}_{1,2}$, obtaining two USp gauge groups. By repeating the various steps in Figure 4 we get an SU(3) gauge theory, where the two conjugate antisymmetric correspond to two extra fundamentals. The SU(3) gauge theory has then two fundamentals and four antifundamentals and it is dual to an LG model (see [9]). Translating the discussion above to the corresponding identities among elliptic genera, we find the expected relation (3.19) in the case of $n=2m$.

The case of SU$(4m+3)$

In order to complete the proof of (3.19) we need to discuss the case SU$(4m+3)$ as well. In this case we get, after m iterations, an SU(3) gauge theory with four fundamentals $\mathbf{Q}_{1,2,3,4}$ with fugacities $(t_1t_2)^{\frac{2m}{3}}u_a$ and two antisymmetric chirals, corresponding to two extra antifundamentals, with fugacities $(t_1t_2)^{\frac{2m}{3}}t_{1,2}$. Again, the SU(3) gauge is dual to an LG theory (see [9]) and we find the expected identity (3.19) in the case of $n = 2m+1$ as well.

This concludes the proof of the identity (3.19). In the end we obtained a model with only Fermi multiplets, as all the chirals T_j in the original gauge theory have been flipped. In the following we will actually use the un-flipped (or partially flipped) duality in order to prove the cases of SU$(2n+1)$ with two antisymemtric chirals and $(4-j, j)$ pairs of fundamentals and antifundamental.

In the next subsections we study the other cases, in which we decrease the number of fundamentals and increase the number of antifundamentals. As a general comment we will see that the derivation in such cases is simpler than above, because the proof does not require an iterative construction. Indeed after deconfining the two antisymmetric tensors and dualizing the unitary and the symplectic gauge nodes we will get SU(N) models with two conjugate antisymmetric and four antifundamental chirals, possibly in addition to a non-trivial set of J-terms. Such models are (modulo charge conjugation) exactly the ones studied so far. It follows that we can use the results of subsection 3.1 and 3.2 in order to prove that the other SU(N) models with two antisymmetric chirals, $n_f < 4$ fundamental chirals and $n_a > 1$ antifundamental chirals are dual to LG theories.

3.3 SU$(2n)$ with three fundamentals and one antifundamental

In this case the dual LG model has five chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

	SU$(2n)$	SU(2)	SU(3)	U$(1)_A$	U$(1)_Q$	U$(1)_{\tilde{Q}}$	U$(1)_{R_0}$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	$\cdot$	1	0	0	0
Q	$\square$	$\cdot$	$\square$	0	1	0	0
$\tilde{Q}$	$\overline{\square}$	$\cdot$	$\cdot$	0	0	1	0
M	$\cdot$	$\cdot$	$\square$	0	1	1	0
T_n	$\cdot$	$\otimes^n_{\text{sym}}\square$	$\cdot$	n	0	0	0
T_{n-1}	$\cdot$	$\otimes^{n-1}_{\text{sym}}\square$	$\overline{\square}$	$n-1$	2	0	0
P_n	$\cdot$	$\otimes^{n-2}_{\text{sym}}\square$	$\square$	n	1	1	0
P_{n-1}	$\cdot$	$\otimes^{n-3}_{\text{sym}}\square$	$\cdot$	$n-1$	3	1	0
Ψ	$\cdot$	$\otimes^{2n-3}_{\text{sym}}\square$	$\cdot$	$-2n+1$	-3	-1	2

(3.26)

The chirals M, T_j, P_j correspond to the following gauge invariant combinations of the charged fields

$$M = Q\tilde{Q}, \quad T_n = A^n\,, \quad T_{n-1} = A^{n-1}Q^2, \quad P_n = A^{n-1}(A\tilde{Q})Q, \quad P_{n-1} = A^{n-2}(A\tilde{Q})Q^3, \tag{3.27}$$

while the J-term is

$$J_\Psi = MT_{n-1}T_n + T_nP_{n-1} + T_{n-1}P_n\,. \tag{3.28}$$

We have computed the 't Hooft anomalies in the two phases and they match. Explicitly we have

$$\begin{aligned} &\kappa_{\mathrm{SU}(2)^2} = \tfrac{n(2n-1)}{2}\,, \qquad && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n-1)\,, \\ &\kappa_{\mathrm{SU}(3)^2} = n\,, && \kappa_{QQ} = -\kappa_{QR_0} = 6n\,, \\ &\kappa_{R_0R_0} = 6n+1\,, && \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R_0} = 2n\,, \\ & && \kappa_{AQ} = \kappa_{A\tilde{Q}} = 0\,. \end{aligned} \tag{3.29}$$

Again, we aim to corroborate the validity of the duality by matching the elliptic genera using other relations already derived in the literature. In this case, differently from the cases above, we will not flip all the chirals in the dual LG description, but only the meson M. We will see that the expected J-term J_Ψ can be reconstructed with our procedure starting from the J-term (3.17) that we expect from the duality of subsection 3.2.

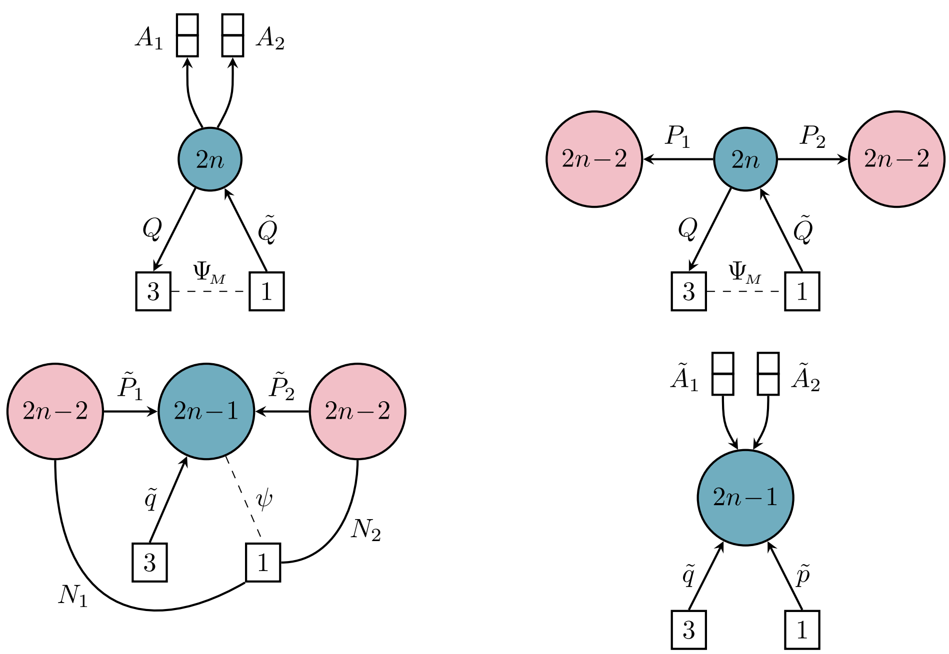

Figure 5: Deconfinement steps of SU($2n$) with two antisymmetrics, three fundamentals and one antifundamental.

The identity that we want to prove is

$$I_{\mathrm{SU}(2n)}^{(3,1,\cdot,2,\cdot)}(\vec{u};v;\cdot;\vec{t};\cdot) = \frac{\prod_{j=0}^{2n-3}\theta\left(q/(t_1^{j+1}t_2^{2n-j-2}u_1u_2u_3v)\right)}{\prod_{a=1}^{3}\theta\left(u_a v\right)\prod_{j=0}^{n-2}\theta\left(t_1^{n-1-j}t_2^{j+1}u_a v\right)} \tag{3.30}$$
$$\times\frac{1}{\prod_{j=0}^{n}\theta\left(t_1^j t_2^{n-j}\right)\cdot\prod_{j=0}^{n-1}\prod_{1\leq a<b\leq 3}\theta\left(t_1^j t_2^{n-j-1}u_a u_b\right)\cdot\prod_{j=0}^{n-3}\theta\left(t_1^{n-2-j}t_2^{j+1}u_1u_2u_3v\right)}\,.$$

In this case, in order to prove this identity through the techniques used above, we will flip the chirals combinations $Q\tilde{Q}$ and $\mathrm{Pf}A_{1,2}$. The first one corresponds to the singlet M in the J-term (3.28), while the other descends from the singlet T_n, and such flip breaks the SU(2) flavor symmetry.

In the following we will redefine the singlets T_n, T_{n-1}, P_n and P_{n-1} as $\mathbf{T}_{n-j,j}$ with $j=0,\ldots,n$, $\mathbf{T}_{n-j-1,j}$ with $j=0,\ldots,n-1$, $\mathbf{P}_{n-j-1,j+1}$ with $j=0,\ldots,n-2$ and $\mathbf{P}_{n-j-2,j+1}$ with $j=0,\ldots,n-3$. Accordingly the Fermi Ψ becomes $\mathbf{\Psi}_{-j-1,-2n+j+2}$ with $j=0,\ldots,2n-3$.

The Fermi flippers of the combinations $Q\tilde{Q}$ and $\mathrm{Pf}A_{1,2}$ give origin to the following J-terms in the original $\mathrm{SU}(2n)$ gauge theory

$$J_{\Psi_M} = Q\tilde{Q}\,,\qquad J_{\Psi_{A_{1,2}}} = \mathrm{Pf}A_{1,2}\,. \tag{3.31}$$

We now proceed by trading the two antisymmetric with two $\mathrm{USp}(2n-2)_{1,2}$ gauge groups, each one connected to the original $\mathrm{SU}(2n)$ gauge group through a bifundamental $P_{1,2}$. In this case the flippers $\Psi_{A_{1,2}}$ disappear and we are left with only J_{Ψ_M}. Observe that in this case the original theory is recovered by dualizing the two symplectic gauge theory using the duality between $\mathrm{USp}(2n)$ with $2n+2$ fundamentals Q and the LG model characterized by the singlets $M=Q^2$ and ψ_0 and J-term $J_{\psi_0} = \mathrm{Pf}\,M$. This duality was originally proposed in [1] and further studied in [7, 32]. We have reviewed the basic aspects of the duality in subsection 2.2.

The $\mathrm{SU}(2n)$ gauge theory has $4n-1$ fundamentals and one antifundamental. It can be dualized according to the rules explained in subsection 2.3, and it gives origin to the third quiver in Figure 5. In this case the J-term is given by $J_\psi = N_2\tilde{P}_2 + N_1\tilde{P}_1$. The last step consists of dualizing the two $\mathrm{USp}(2n-2)$ groups, each one with $2n$ chiral fundamentals to an LG model. The final quiver is the fourth one in figure 5. In this case there are two J-terms associated to two Fermi singlets $\Psi_{1,2}^{(0)}$. They read $J_{\Psi_{1,2}^{(0)}} = \tilde{A}_{1,2}^{n-1}\tilde{p}$.

We ended up with an $\mathrm{SU}(2n-1)$ gauge theory with four antifundamentals and two conjugate antisymmetric. This is, up to a conjugation, the same field content of the confining duality studied in subsection 3.2. In addition we have the non-vanshing J-terms $J_{\Psi_{1,2}^{(0)}}$.

The theory is then dual to an LG model where the J-term (3.17) is modified by symmetry breaking pattern implied by the presence of $J_{\Psi_{1,2}^{(0)}}$.

In order to take such breaking into account let us consider an SU$(2N+1)$ gauge theory with four antifundamentals $\tilde{Q} = \{\tilde{q}_1, \tilde{q}_2, \tilde{q}_3, \tilde{p}\}$ and two conjugate antisymmetric tensors $\tilde{A} = \{\tilde{A}_1, \tilde{A}_2\}$. The singlets of the unbroken phase are $\tilde{T}_N$ and $\tilde{T}_{N-1}$ and the 2d superpotential[10]

$$W_{2d} = \sum_{j=0}^{2N-1} \sum_{k=0}^{N} \sum_{\ell=0}^{N-1} |\epsilon_{abcd}| \psi_{j-2N+1,-j} \tilde{T}_{k,N-k}^{(a)} \tilde{T}_{\ell,N-1-\ell}^{(bcd)} \delta_{j+k+\ell,2N-1}\,, \tag{3.32}$$

where we used the notations of 3.2 for the SU(2) and SU(4) indices. Then we fix $N = n-1$ and consider the effect of $J_{\Psi_{1,2}^{(0)}}$. The singlets $\tilde{T}_{n-1}$ and $\tilde{T}_{n-2}$ are split as

$$\begin{aligned} \tilde{T}_{j,n-1-j}^{(a)} &= \tilde{A}_1^j \tilde{A}_2^{n-1-j}\ \tilde{q}_a\,, & \tilde{T}_{j,n-1-j}^{(0)} &= \tilde{A}_1^j \tilde{A}_2^{n-1-j}\ \tilde{R}_-\,, \\ \tilde{T}_{j,n-2-j}^{(0)} &= \tilde{A}_1^j \tilde{A}_2^{n-2-j}\ \tilde{q}_1\tilde{q}_2\tilde{q}_3\,, & \tilde{T}_{j,n-2-j}^{(ab)} &= \tilde{A}_1^j \tilde{A}_2^{n-2-j}\ \tilde{q}_a\tilde{q}_b\tilde{R}_-\,. \end{aligned} \tag{3.33}$$

In this way the superpotential of the LG for the deformed theory is

$$\begin{aligned} W_{2d} = &\sum_{j=0}^{2n-3} \sum_{k=0}^{n-1} \sum_{\ell=0}^{n-2} \psi_{j-2n+3,-j} (|\epsilon_{abc}| \tilde{T}_{k,n-1-k}^{(a)} \tilde{T}_{\ell,n-2-\ell}^{(bc)} + \tilde{T}_{k,n-1-k}^{(0)} \tilde{T}_{\ell,n-2-\ell}^{(0)}) \delta_{j+k+\ell,2n-3} \\ &+ \psi_1^{(0)} \tilde{T}_{n-1,0}^{(0)} + \psi_2^{(0)} \tilde{T}_{0,n-1}^{(0)}, \end{aligned} \tag{3.34}$$

where the deformation in the second line can be integrated out using the equations of motion.

The superpotential (3.34) is equivalent to the one that can be read from the J-term (3.28) once the dictionary between the singlets $\tilde{T}$ and the singlets (3.27) is specified and after setting the combinations $Q\tilde{Q}$ and Pf$A_{1,2}$ in (3.28) to zero.

$$\begin{aligned} &\mathbf{T}_{n-1-j,\,j+1} = \tilde{T}_{j,\,n-2-j}^{(0)}\,, & &\mathbf{T}_{n-1-j,\,j}^{(ab)} = |\epsilon^{ab}{}_c| \tilde{T}_{j,\,n-1-j}^{(c)}\,, \\ &\mathbf{P}_{n-1-j,\,j+1}^{(a)} = |\epsilon^{a}{}_{bc}| \tilde{T}_{j,\,n-1-j}^{(bc)}\,, & &\mathbf{P}_{n-1-j,\,j} = \tilde{T}_{j\,,n-1-j}^{(0)}\,, \\ &\Psi_{-j-1,\,j-2n+2} = \psi_{j-2n+3,\,-j} \end{aligned} \tag{3.35}$$

The same strategy explained in the field theory discussion can be applied at the level of the elliptic genus. In this case the four steps in Figure 5 can be reproduced at the level of the elliptic genus as in the previous cases of SU(N) with two antisymmetric and four fundamentals. The last step corresponds to plugging the identity (3.19) inside the elliptic genus of the SU$(2n-1)$ gauge theory. In this way we obtain the relation (3.30) as expected.

[10] Here we use the notion of 2d superpotential instead of the one of J-terms because the structure of the former is more intuitive

3.4 $\mathrm{SU}(2n+1)$ with three fundamentals and one antifundamental

In this case the dual LG model has five chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the following table

	$\mathrm{SU}(2n+1)$	$\mathrm{SU}(2)$	$\mathrm{SU}(3)$	$\mathrm{U}(1)_A$	$\mathrm{U}(1)_Q$	$\mathrm{U}(1)_{\tilde{Q}}$	$\mathrm{U}(1)_{R_0}$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	$\cdot$	1	0	0	0
Q	$\square$	$\cdot$	$\square$	0	1	0	0
$\tilde{Q}$	$\overline{\square}$	$\cdot$	$\cdot$	0	0	1	0
M	$\cdot$	$\cdot$	$\square$	0	1	1	0
T_n	$\cdot$	$\otimes_{\mathrm{sym}}^{n}\square$	$\square$	n	1	0	0
T_{n-1}	$\cdot$	$\otimes_{\mathrm{sym}}^{n-1}\square$	$\cdot$	$n-1$	3	0	0
P_{n+1}	$\cdot$	$\otimes_{\mathrm{sym}}^{n-1}\square$	$\cdot$	$n+1$	0	1	0
P_n	$\cdot$	$\otimes_{\mathrm{sym}}^{n-2}\square$	$\overline{\square}$	n	2	1	0
Ψ	$\cdot$	$\otimes_{\mathrm{sym}}^{2n-2}\square$	$\cdot$	$-2n$	-3	-1	2

(3.36)

We have computed the 't Hooft anomalies in the two phases and they match. Explicitly:

$$\begin{aligned} &\kappa_{\mathrm{SU}(2)^2} = \tfrac{n(2n+1)}{2}\,, \\ &\kappa_{\mathrm{SU}(3)^2} = \tfrac{2n+1}{2}\,, \\ &\kappa_{R_0R_0} = 6n+4\,, \end{aligned} \qquad \begin{aligned} &\kappa_{AA} = -\kappa_{AR_0} = 2n(2n+1)\,, \\ &\kappa_{QQ} = -\kappa_{QR_0} = 6n+3\,, \\ &\kappa_{\tilde{Q}\tilde{Q}} = \kappa_{\tilde{Q}R_0} = 2n+1\,, \\ &\kappa_{AQ} = \kappa_{A\tilde{Q}} = 0\,. \end{aligned} \tag{3.37}$$

The chirals M, T_j and P_j correspond to the following gauge invariant combinations of the charged fields

$$M = Q\tilde{Q}\,, \quad T_n = A^nQ\,, \quad T_{n-1} = A^{n-1}Q^3\,, \quad P_{n+1} = A^n(A\tilde{Q})\,, \quad P_n = A^{n-1}(A\tilde{Q})Q^2 \tag{3.38}$$

while the J-term is

$$J_\Psi = MT_n^2 + T_{n-1}P_{n+1} + T_nP_n\,. \tag{3.39}$$

Again we aim to corroborate the validity of the duality by matching the elliptic genera using other relations already derived in the literature. In this case we will flip the meson M and the combinations $A_1^nQ_1$ and $A_2^nQ_2$ in the dual LG and we show that the expected J-term in (3.39) can be reconstructed by our procedure using the J-term expected from the duality of subsection 3.1. The derivation is very similar to the one discussed in subsection 3.3, and for this reason we will omit many technical details in this case.

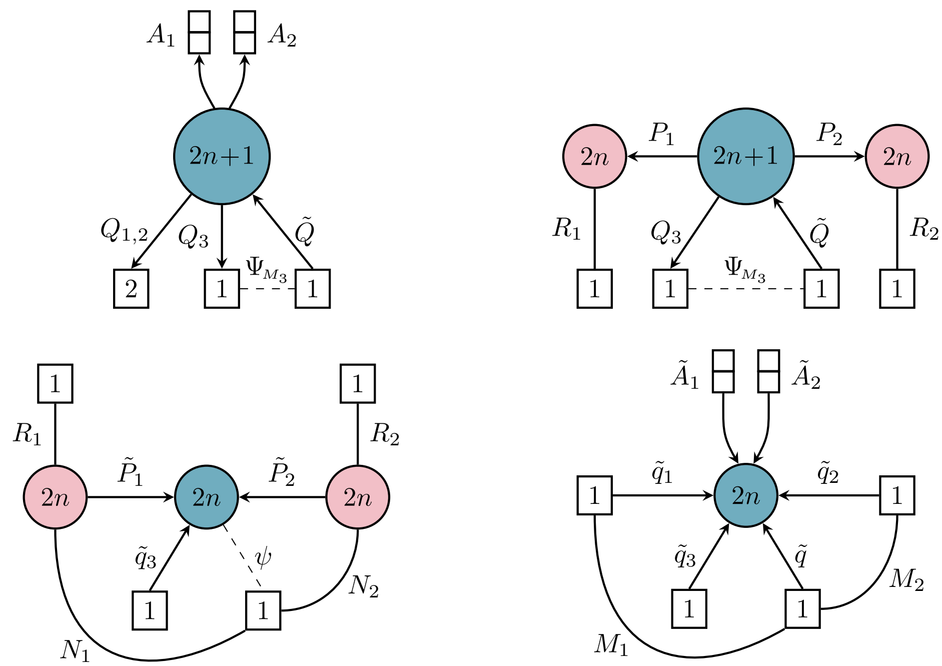

Figure 6: Deconfinement steps of $\mathrm{SU}(2n+1)$ with two antisymmetrics, three fundamentals and one antifundamental.

The identity we want is

$$I^{(3,1,\cdot,2,\cdot)}_{\mathrm{SU}(2n+1)}(\vec{u};v;\cdot;\vec{t};\cdot) = \frac{\prod_{j=0}^{2n-2}\theta(q/(t_1^{j+1}t_2^{2n-j-1}u_1u_2u_3v))}{\prod_{a=1}^{3}\theta(u_av)\prod_{j=0}^{n}\prod_a\theta(t_1^jt_2^{n-j}u_a)} \\ \times \frac{1}{\prod_{j=0}^{n-1}(\theta(t_1^jt_2^{n-j-1}u_1u_2u_3)\theta(t_1^{n-j}t_2^{j+1}v))\prod_{j=0}^{n-2}\prod_{a<b}\theta(t_1^{n-j-1}t_2^{j+1}u_au_bv)}\,. \tag{3.40}$$

In order to prove this relation we deconfine the two antisymmetric as in Figure 6, obtaining two $\mathrm{USp}(2n)$ gauge nodes, two bifundamentals $P_{1,2}$ and two new fundamentals $R_{1,2}$. The original antisymmetric chirals $A_{1,2}$ correspond to the combinations $P_{1,2}^2$. Furthermore, two out of the three original fundamentals $Q_{1,2}$ correspond in this quiver to the composites $P_{1,2}R_{1,2}$. The J-term flipping the operator M splits here as

$$J_{\Psi_{M_1}} = \tilde{Q}P_1R_1\,, \quad J_{\Psi_{M_2}} = \tilde{Q}P_2R_2\,, \quad J_{\Psi_{M_3}} = \tilde{Q}Q_3\,, \tag{3.41}$$

signalling the explicit breaking of the $\mathrm{SU}(3)$ flavor symmetry in the process.

The $\mathrm{SU}(2n+1)$ gauge theory has $4n+1$ fundamentals and one antifundamental. It can be dualized according to the rules explained in subsection 2.3, and it gives origin to the third quiver in Figure 6. In this case the new J-term generated by the duality is given by $J_\psi = N_2\tilde{P}_2 + N_1\tilde{P}_1$. Furthermore, the two J-terms for $\Psi_{M_{1,2}}$ become $J_{\Psi_{M_{1,2}}} = \tilde{Q}N_{1,2}$. The last step consists of dualizing the two USp$(2n)$ groups, each one with $2n+2$ chiral fundamentals to an LG. The final quiver is the fourth one in Figure 6. In this case there are two J-terms associated to two Fermi singlets $\Psi^{(0)}_{1,2}$. They read $J_{\Psi^{(0)}_1} = \tilde{A}_1^{n-1}\tilde{q}_1\tilde{q}$ and $J_{\Psi^{(0)}_2} = \tilde{A}_2^{n-1}\tilde{q}_2\tilde{q}$.

Again we are in the situation described in subsection 3.3, where after deconfining the tensors, dualizing the unitary gauge group and confining the tensors back, we have, up to an overall charge conjugation, a model already described in 3.1. We can the use the duality of such model with an LG one in order to prove the validity of (3.40) and to reconstruct (up to flippers) the expected J-term (3.39). The details of the derivation are straightforward and we leave them to the interested reader.

3.5 SU$(2n)$ with two fundamentals and two antifundamentals

In this case the dual LG model has seven chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

	SU$(2n)$	SU(2)	SU(2)	SU(2)	U$(1)_A$	U$(1)_Q$	U$(1)_{\tilde{Q}}$	U$(1)_{R_0}$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	$\cdot$	$\cdot$	1	0	0	0
Q	$\square$	$\cdot$	$\square$	$\cdot$	0	1	0	0
$\tilde{Q}$	$\overline{\square}$	$\cdot$	$\cdot$	$\square$	0	0	1	0
M	$\cdot$	$\cdot$	$\square$	$\square$	0	1	1	0
B	$\cdot$	$\square$	$\cdot$	$\cdot$	1	0	2	0
T_n	$\cdot$	$\otimes^n_{\text{sym}}\square$	$\cdot$	$\cdot$	n	0	0	0
T_{n-1}	$\cdot$	$\otimes^{n-1}_{\text{sym}}\square$	$\cdot$	$\cdot$	$n-1$	2	0	0
P_n	$\cdot$	$\otimes^{n-2}_{\text{sym}}\square$	$\square$	$\square$	n	1	1	0
P_{n+1}	$\cdot$	$\otimes^{n-3}_{\text{sym}}\square$	$\cdot$	$\cdot$	$n+1$	0	2	0
R	$\cdot$	$\otimes^{n-4}_{\text{sym}}\square$	$\cdot$	$\cdot$	n	2	2	0
Ψ	$\cdot$	$\otimes^{2n-4}_{\text{sym}}\square$	$\cdot$	$\cdot$	$-2n$	-2	-2	2

(3.42)

The chirals B, M, T_j, P_j and R correspond to the following gauge invariant combinations of the charged field

$$\begin{aligned} &T_n = A^n\,, && P_n = A^{n-1}(A\tilde{Q})Q\,, && M = Q\tilde{Q}\,, && R = A^{n-2}(A\tilde{Q})^2Q^2\,,\\ &T_{n-1} = A^{n-1}Q^2\,, && P_{n+1} = A^{n-1}(A\tilde{Q})^2\,, && B = A\tilde{Q}^2\,, && \end{aligned} \tag{3.43}$$

while the J-term is

$$J_\Psi = M^2T_n^2 + MP_nT_n + T_nR + T_{n-1}P_{n+1} + P_n^2 + BT_{n-1}T_n\,. \tag{3.44}$$

The 't Hooft anomalies match in the two phases and are explicitly

$$\begin{aligned}
&\kappa_{\mathrm{SU}(2)^2_A} = \tfrac{n(2n-1)}{2}\,, && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n-1)\,, \\
&\kappa_{\mathrm{SU}(2)^2_Q} = \kappa_{\mathrm{SU}(2)_{\tilde{Q}^2}} = n\,, && \kappa_{QQ} = -\kappa_{QR_0} = \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R_0} = 4n\,, \\
&\kappa_{R_0R_0} = 6n+1\,, && \kappa_{AQ} = \kappa_{A\tilde{Q}} = 0\,.
\end{aligned} \tag{3.45}$$

In the following we show the matching of the elliptic genera along the same lines of the discussion performed in the other examples. In this case we will flip the meson M and the baryons B, $\mathrm{Pf}A_{1,2}$ in the dual LG, and we will show that the expected J-term in (3.44) can be reconstructed by our procedure using the J-term expected from the duality of subsection 3.1.

At the level of the elliptic genus the identity associated to this duality is

$$\begin{aligned}
I^{(2,2,\cdot,2,\cdot)}_{\mathrm{SU}(2n)}(\vec{u};\vec{v};\cdot;\vec{t};\cdot) &= \frac{\prod_{j=0}^{2n-4}\theta(q/(t_1^{j+2}t_2^{2n-j-2}u_1u_2v_1v_2))}{\prod_{a,b=1}^{2}\theta(u_av_b)\theta(t_{1,2}v_1v_2)\prod_{j=0}^{n}\theta(t_1^jt_2^{n-j})\prod_{j=0}^{n-1}\theta(t_1^{n-1-j}t_2^ju_1u_2)} \\
&\times\frac{1}{\prod_{j=0}^{n-4}\theta(t_1^{n-j-2}t_2^{j+2}u_1u_2v_1v_2)\prod_{j=0}^{n-3}\theta(t_1^{n-j-1}t_2^{j+2}v_1v_2)\prod_{j=0}^{n-2}\prod_{a,b=1}^{2}\theta(t_1^{n-j-1}t_2^{j+1}u_av_b)}\,.
\end{aligned} \tag{3.46}$$

In order to prove this relation we deconfine the two antisymmetric as in Figure 7, obtaining two $\mathrm{USp}(2n-2)$ gauge nodes, with two bifundamentals $P_{1,2}$. The original antisymmetric chirals $A_{1,2}$ correspond to the combinations $P_{1,2}^2$. The J-terms at this stage are the ones that flip the operators M and B

$$J_{\Psi_M} = \tilde{Q}Q\,,\quad J_{\Psi_{B_1}} = \tilde{Q}^2P_1^2\,,\quad J_{\Psi_{B_2}} = \tilde{Q}^2P_2^2\,. \tag{3.47}$$

The $\mathrm{SU}(2n)$ gauge theory has $4n-2$ fundamentals and two antifundamentals. It can be dualized according to the rules explained in subsection 2.3, giving origin to the third quiver in Figure 7. In this case the new J-term generated by the duality is $J_\psi = N_2\tilde{P}_2 + N_1\tilde{P}_1$. In addition, the two J-terms for Ψ_{B_1} and Ψ_{B_2} become $J_{\Psi_{B_1}} = N_1^2$. and $J_{\Psi_{B_2}} = N_2^2$ respectively.

The last step consists in dualizing the two $\mathrm{USp}(2n-2)$ groups, each one with $2n$ chiral fundamentals, to an LG model. The final quiver is the fourth one in Figure 7. In this case there are two J-terms associated to two Fermi singlets $\Psi^{(0)}_{1,2}$. They read $J_{\Psi^{(0)}_{1,2}} = \tilde{A}_{1,2}^{n-2}\tilde{p}^2$.

Similarly to the cases discussed above we have, up to an overall charge conjugation, a model already described in 3.1. We can the use the duality of such model with an LG one in order to prove the validity of (3.46) and to reconstruct (up to flippers) the expected J-term (3.44). The details of the derivation are straightforward and we leave them to the interested reader.

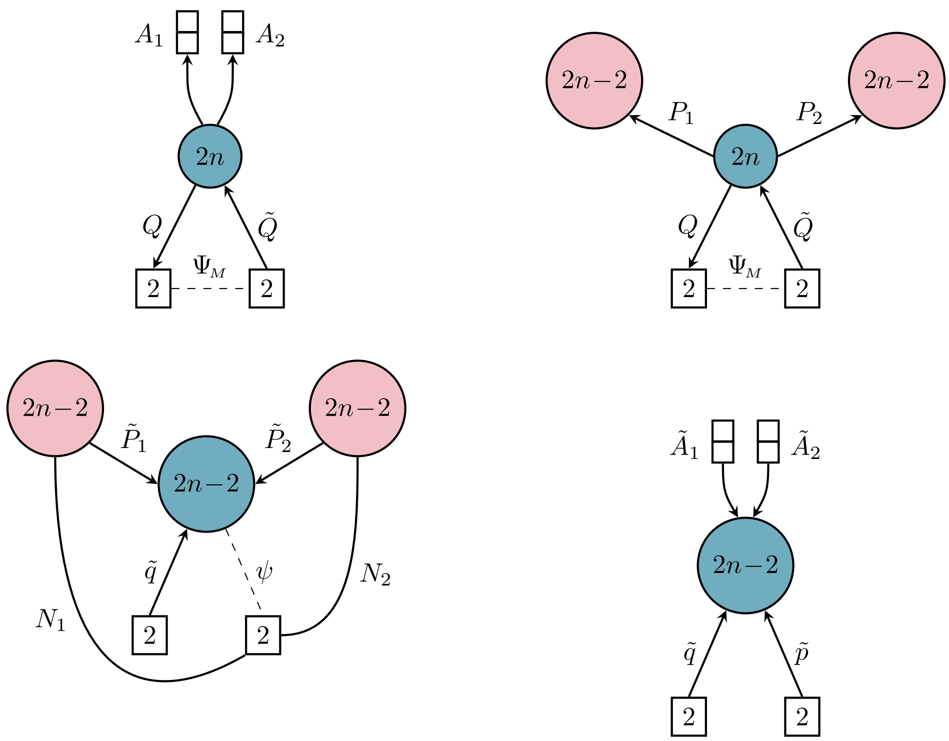

Figure 7: Deconfinement steps of SU($2n$) with two antisymmetrics, two fundamentals and two antifundamentals.

3.6 SU($2n+1$) with two fundamentals and two antifundamentals

In this case the dual LG model has six chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

	SU($2n+1$)	SU(2)	SU(2)	SU(2)	U(1)$_A$	U(1)$_Q$	U(1)$_{\tilde{Q}}$	U(1)$_{R_0}$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	$\cdot$	$\cdot$	1	0	0	0
Q	$\square$	$\cdot$	$\square$	$\cdot$	0	1	0	0
$\tilde{Q}$	$\overline{\square}$	$\cdot$	$\cdot$	$\square$	0	0	1	0
M	$\cdot$	$\cdot$	$\square$	$\square$	0	1	1	0
B	$\cdot$	$\square$	$\cdot$	$\cdot$	1	0	2	0
T_n	$\cdot$	$\otimes^{n}_{\text{sym}}\square$	$\square$	$\cdot$	n	1	0	0
T_{n-1}	$\cdot$	$\otimes^{n-3}_{\text{sym}}\square$	$\square$	$\cdot$	$n+1$	1	2	0
P_n	$\cdot$	$\otimes^{n-1}_{\text{sym}}\square$	$\cdot$	$\square$	$n+1$	0	1	0
P_{n-1}	$\cdot$	$\otimes^{n-2}_{\text{sym}}\square$	$\cdot$	$\square$	n	2	1	0
Ψ	$\cdot$	$\otimes^{2n-3}_{\text{sym}}\square$	$\cdot$	$\cdot$	$-2n-1$	-2	-2	2

(3.48)

The chirals T_j, P_j correspond to the following gauge invariant combinations of the charged fields

$$T_n = A^n Q\,, \quad T_{n-1} = A^{n-1}(A\tilde{Q})^2 Q\,, \quad P_n = A^n(A\tilde{Q})\,, \quad P_{n-1} = A^{n-1}(A\tilde{Q})Q^2\,. \quad (3.49)$$

In addition there are a meson $M = Q\tilde{Q}$ and the baryons $B_{1,2} = A_{1,2}\tilde{Q}^2$, while the expected J-term compatible with the global symmetry is

$$J_\Psi = M T_n P_n + B T_n^2 + T_n T_{n-1} + P_n P_{n-1}\,. \quad (3.50)$$

We have matched the 't Hooft anomalies in the two phases. Explicitly we have

$$\begin{aligned}
&\kappa_{\mathrm{SU}(2)^2_A} = \tfrac{n(2n+1)}{2}\,, && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n+1)\,,\\
&\kappa_{\mathrm{SU}(2)^2_Q} = \kappa_{\mathrm{SU}(2)_{\tilde{Q}^2}} = \tfrac{2n+1}{2}\,, && \kappa_{QQ} = -\kappa_{QR_0} = \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R_0} = 4n+2\,,\\
&\kappa_{R_0R_0} = 6n+4\,, && \kappa_{AQ} = \kappa_{A\tilde{Q}} = 0\,.
\end{aligned} \quad (3.51)$$

In the following we show the matching of the elliptic genera along the same lines of the discussion performed in the other examples. In this case we will flip the combinations $Q\tilde{Q}$, $A_{1,2}\tilde{Q}^2$ and $A_{1,2}^n Q$ and we will show that the expected J-term in (3.50) can be reconstructed by our procedure starting from the J-term expected from the duality of subsection 3.2.

At the level of the elliptic genus the identity associated to this duality is

$$\begin{aligned}
I^{(2,2,\cdot,2,\cdot)}_{\mathrm{SU}(2n+1)}(\vec{u};\vec{v};\cdot;\vec{t};\cdot) &= \frac{\prod_{j=0}^{2n-3}\theta(q/(t_1^{j+2}t_2^{2n-j-1}u_1u_2v_1v_2))}{\theta(t_{1,2}v_1v_2)\prod_{a=1}^{2}(\prod_{j=0}^{n}\theta(t_1^j t_2^{n-j}u_a)\cdot\prod_{j=0}^{n-1}\theta(t_1^{n-j}t_2^{j+1}v_a))}\\
&\times\frac{1}{\prod_{a,b=1}^{2}\theta(u_av_b)\prod_{a=1}^{2}(\prod_{j=0}^{n-3}\theta(t_1^{n-1-j}t_2^{j+2}u_av_1v_2)\cdot\prod_{j=0}^{n-2}\prod_{a=1}^{2}\theta(t_1^{n-1-j}t_2^{j+1}u_1u_2v_a))}\,.
\end{aligned} \quad (3.52)$$

In order to prove this relation we deconfine the two antisymmetric as in Figure 8, obtaining two $\mathrm{USp}(2n)$ gauge nodes, with two bifundamentals $P_{1,2}$ and two fundamentals $R_{1,2}$. The original antisymmetric chirals $A_{1,2}$ correspond to the combinations $P_{1,2}^2$ while the original fundamentals $Q_{1,2}$ correspond to the combinations $P_{1,2}R_{1,2}$. The J-terms at this stage are the ones flipping the operators M and B and they are

$$J_{\Psi_{M_a}} = \tilde{Q}P_aR_a\,, \quad J_{\Psi_{B_a}} = \tilde{Q}^2P_a^2\,, \quad (3.53)$$

for $a = 1,2$. The $\mathrm{SU}(2n+1)$ gauge theory has $4n$ fundamentals and two antifundamentals. It can be dualized according to the rules explained in subsection 2.3, and it gives origin to the third quiver in Figure 8. In this case the new J-term generated by

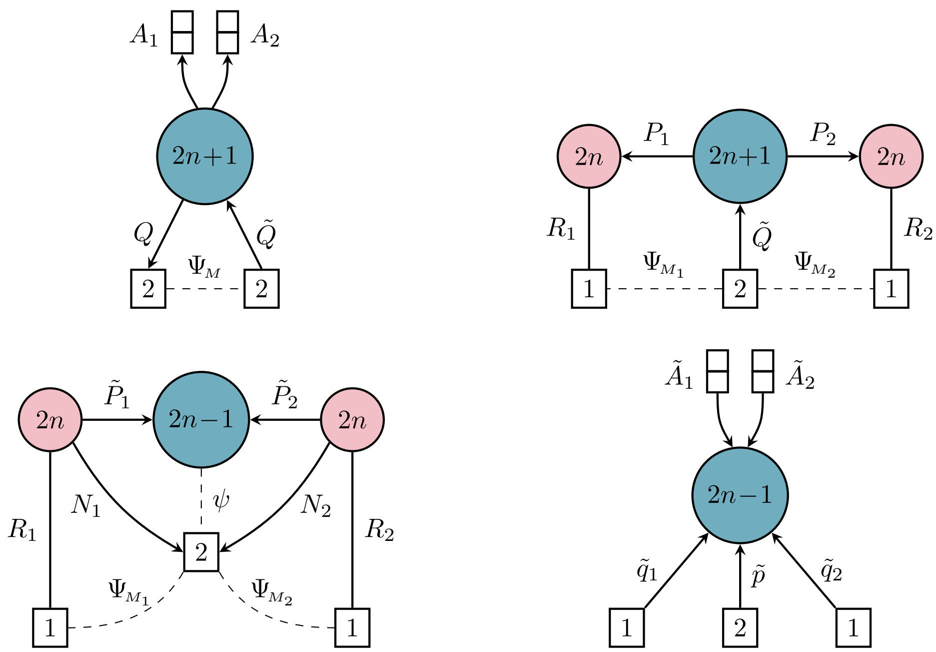

Figure 8: Deconfinement steps of $\mathrm{SU}(2n+1)$ with two antisymmetrics, two fundamentals and two antifundamentals.

the duality is $J_\psi = N_2 \tilde{P}_2 + N_1 \tilde{P}_1$. In addition the J-terms for Ψ_{M_a} and Ψ_{B_a} become $J_{\Psi_{M_a}} = R_a N_a$ and $J_{\Psi_{B_a}} = N_a^2$.

The last step consists of dualizing the two $\mathrm{USp}(2n)$ groups, each one with $2n+2$ chiral fundamentals, to an LG. The final quiver is the fourth one in Figure 8. In this case there are two J-terms associated to two Fermi singlets $\Psi_{1,2}^{(0)}$. They read $J_{\Psi_{1,2}^{(0)}} = \tilde{A}_{1,2}^{n-2} \tilde{q}_{1,2} \tilde{p}^2$.

Similarly to the cases discussed above we have, up to an overall charge conjugation, a model already described in 3.2. We can the use the duality of such model with an LG in order to prove the validity of (3.52) and to reconstruct (up to flippers) the expected J-term (3.50). The details of the derivation are straightforward and we leave them to the interested reader.

3.7 $\mathrm{SU}(2n)$ with one fundamentals and three antifundamentals

In this case the dual LG model has six chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

	$SU(2n)$	$SU(2)$	$SU(3)$	$U(1)_A$	$U(1)_Q$	$U(1)_{\tilde{Q}}$	$U(1)_{R_0}$
A	$\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	$\square$	$\cdot$	1	0	0	0
Q	$\square$	$\cdot$	$\cdot$	0	1	0	0
$\tilde{Q}$	$\overline{\square}$	$\cdot$	$\square$	0	0	1	0
M	$\cdot$	$\cdot$	$\square$	0	1	1	0
B	$\cdot$	$\square$	$\overline{\square}$	1	0	2	0
T_n	$\cdot$	$\otimes^n_{\text{sym}}\square$	$\cdot$	n	0	0	0
P_n	$\cdot$	$\otimes^{n-2}_{\text{sym}}\square$	$\square$	n	1	1	0
P_{n+1}	$\cdot$	$\otimes^{n-3}_{\text{sym}}\square$	$\overline{\square}$	$n+1$	0	2	0
Π_{n+1}	$\cdot$	$\otimes^{n-5}_{\text{sym}}\square$	$\cdot$	$n+1$	1	3	0
Ψ	$\cdot$	$\otimes^{2n-5}_{\text{sym}}\square$	$\cdot$	$-2n-1$	-1	-3	2

(3.54)

The chirals T_j, P_j correspond to the following gauge invariant combinations of the charged fields

$$T_n = A^n\,,\quad P_n = A^{n-1}(A\tilde{Q})Q\,,\quad P_{n+1} = A^{n-1}(A\tilde{Q})^2\,,\quad \Pi_{n+1} = A^{n-2}(A\tilde{Q})^3Q\,, \tag{3.55}$$

In addition there are a meson $M = \tilde{Q}Q$ and the baryons $B_{1,2} = A_{1,2}\tilde{Q}^2$, while the expected J-term compatible with the global symmetry is

$$J_\Psi = BT_nP_n + P_nP_{n+1} + T_n\Pi_{n+1} + MT_nP_{n+1}\,. \tag{3.56}$$

We checked the matching of the 't Hooft anomalies in the two theories. Explicitly we have

$$\begin{aligned} &\kappa_{SU(2)^2} = \tfrac{n(2n-1)}{2}\,, && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n-1)\,,\\ &\kappa_{SU(3)} = n\,, && \kappa_{QQ} = -\kappa_{QR_0} = 2n\,,\\ &\kappa_{R_0R_0} = 6n+1\,, && \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R_0} = 6n\,,\\ & && \kappa_{AQ} = \kappa_{A\tilde{Q}} = 0\,. \end{aligned} \tag{3.57}$$

In the following we show the matching of the eblliptic genera along the same lines of the discussion performed in the other examples. In this case we will flip the meson M and the baryons $B_{1,2}$ and $\mathrm{Pf}A_{1,2}$ in the dual LG and we will show that the expected J-term in (3.56) can be reconstructed by our procedure using the J-term expected from the duality of subsection 3.2. At the level of the elliptic genus the identity associated to this duality is

$$I^{(1,3,\cdot,2,\cdot)}_{SU(2n)}(u;\vec{v};\cdot;\vec{t};\cdot) = \frac{\prod_{j=0}^{2n-5}\theta(q/(t_1^{j+3}t_2^{2n-j-1}uv_1v_2v_3))}{\prod_a\theta(uv_a)\prod_{a<b}(\theta(t_{1,2}v_av_b))\prod_{j=0}^{n-3}\theta(t_1^{n-1-j}t_2^{j+2}v_av_b)}$$

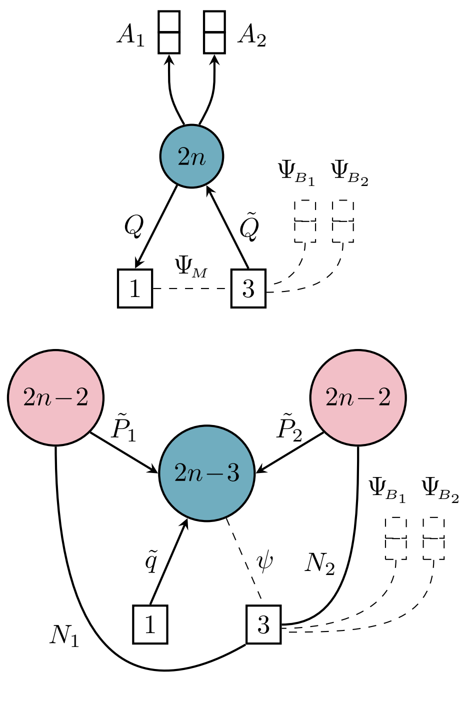

Figure 9: Deconfinement steps of SU($2n$) with two antisymmetrics, one fundamental and three antifundamentals.

$$\times \frac{1}{\prod_{j=0}^{n-5} \theta(t_1^{n-j-2} t_2^{j+3} uv_1v_2v_3) \prod_{j=0}^{n} \theta(t_1^{n-j} t_2^{j}) \prod_{j=0}^{n-2} \prod_a \theta(t_1^{n-1-j} t_2^{j+1} uv_a)} . \qquad (3.58)$$

In order to prove this relation we deconfine the two antisymmetric as in Figure 9, obtaining two USp($2n-2$) gauge nodes, with two bifundamentals $P_{1,2}$. The original antisymmetric chirals $A_{1,2}$ correspond to the combinations $P_{1,2}^2$. The J-term at this stage are the ones flipping the operators M and B and they read

$$J_{\Psi_M} = \tilde{Q}Q\,, \quad J_{\Psi_{B_{1,2}}} = Q^2 P_{1,2}^2\,. \qquad (3.59)$$

The SU($2n$) gauge theory has $4n-1$ fundamentals and one antifundamental. It can be dualized according to the rules explained in subsection 2.3, and it gives origin to the third quiver in Figure 9.

In this case the new J-term generated by the duality is $J_\psi = N_2\tilde{P}_2 + N_1\tilde{P}_1$. In addition the J-terms for Ψ_{B_1} and Ψ_{B_2} become $J_{\Psi_{B_1}} = N_1^2$ and $J_{\Psi_{B_2}} = N_2^2$ respectively.

The last step consists of dualizing the two USp($2n-2$) groups, each one with $2n$ chiral fundamentals, to an LG. The final quiver is the fourth one in Figure 9. In this case there are two J-terms associated to two Fermi singlets $\Psi_{1,2}^{(0)}$. They read $J_{\Psi_{1,2}^{(0)}} = \tilde{A}_{1,2}^{n-3}\tilde{p}^3$.

Again we have, up to an overall charge conjugation, a model already described in 3.2. We can the use the duality of such model with an LG in order to prove the validity of (3.58) and to reconstruct (up to flippers) the expected J-term (3.56). The details of the derivation are straightforward and we leave them to the interested reader.

3.8 $\mathrm{SU}(2n+1)$ with one fundamentals and three antifundamentals

In this case the dual LG model has six chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

$$\begin{array}{c|c|c|c|c|c|c|c|}
 & \mathrm{SU}(2n+1) & \mathrm{SU}(2) & \mathrm{SU}(3) & \mathrm{U}(1)_A & \mathrm{U}(1)_Q & \mathrm{U}(1)_{\tilde{Q}} & \mathrm{U}(1)_{R_0} \\
\hline
A & \begin{smallmatrix}\square\\ \square\end{smallmatrix} & \square & \cdot & 1 & 0 & 0 & 0 \\
Q & \square & \cdot & \cdot & 0 & 1 & 0 & 0 \\
\tilde{Q} & \overline{\square} & \cdot & \square & 0 & 0 & 1 & 0 \\
\hline
M & \cdot & \cdot & \square & 0 & 1 & 1 & 0 \\
B & \cdot & \square & \overline{\square} & 1 & 0 & 2 & 0 \\
T_n & \cdot & \otimes^n_{\mathrm{sym}}\square & \cdot & n & 1 & 0 & 0 \\
P_{n+1} & \cdot & \otimes^{n-1}_{\mathrm{sym}}\square & \square & n+1 & 0 & 1 & 0 \\
P_{n+2} & \cdot & \otimes^{n-4}_{\mathrm{sym}}\square & \cdot & n+2 & 0 & 3 & 0 \\
R & \cdot & \otimes^{n-3}_{\mathrm{sym}}\square & \overline{\square} & n+1 & 1 & 2 & 0 \\
\Psi & \cdot & \otimes^{2n-4}_{\mathrm{sym}}\square & \cdot & -2n-2 & -1 & -3 & 2 \\
\end{array} \tag{3.60}$$

The chirals T_j, P_j and R correspond to the following gauge invariant combinations of the charged fields

$$T_n = A^n Q\,, \quad P_{n+1} = A^n(A\tilde{Q})\,, \quad P_{n+2} = A^{n-1}(A\tilde{Q})^3\,, \quad R = A^{n-1}(A\tilde{Q})^2 Q\,, \tag{3.61}$$

In additions there are other singlets identified with the meson $M = Q\tilde{Q}$ and the baryons $B_{1,2} = A_{1,2}\tilde{Q}^2$. The J-term compatible with the global symmetry is

$$J_\Psi = MP_{n+1}^2 + BP_{n+1}T_n + T_nP_{n+2} + P_{n+1}R\,. \tag{3.62}$$

We checked the matching of the 't Hooft anomalies in the two theories. Explicitly we have

$$\begin{aligned}
&\kappa_{\mathrm{SU}(2)^2} = \tfrac{n(2n+1)}{2}\,, && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n+1)\,,\\
&\kappa_{\mathrm{SU}(3)^2} = \tfrac{2n+1}{2} && \kappa_{QQ} = -\kappa_{QR_0} = 2n+1\,,\\
&\kappa_{R_0R_0} = 6n+4\,, && \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R_0} = 6n+3\,,\\
& && \kappa_{AQ} = \kappa_{A\tilde{Q}} = 0\,.
\end{aligned} \tag{3.63}$$

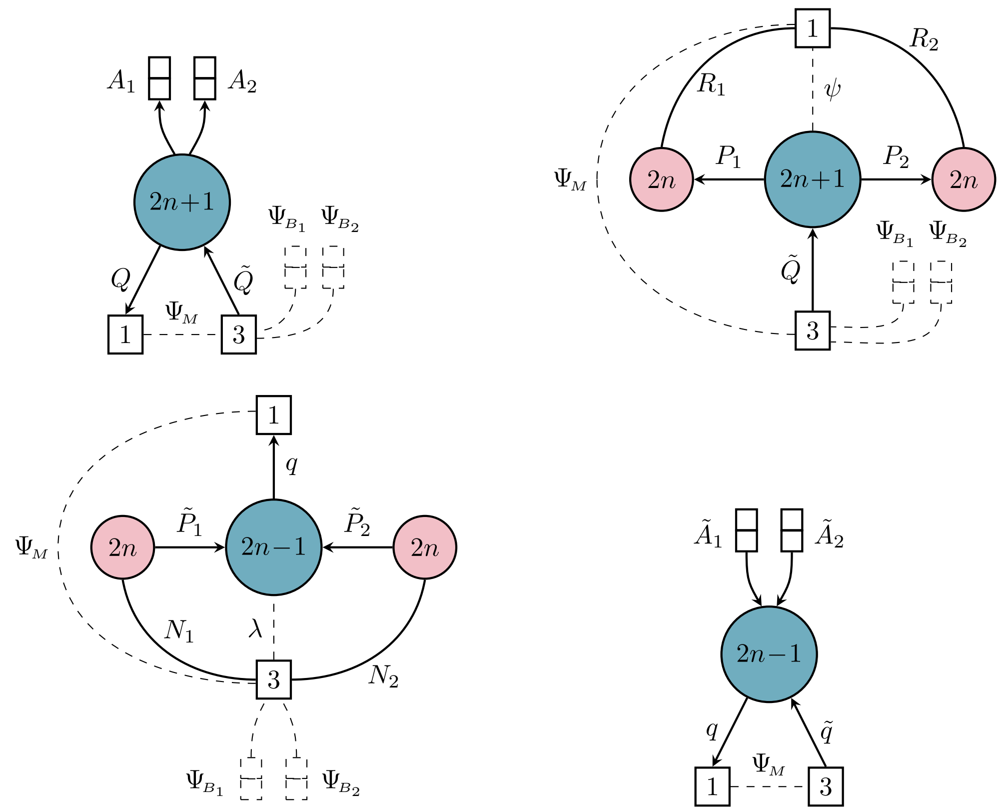

Figure 10: Deconfinement steps of $\mathrm{SU}(2n+1)$ with two antisymmetrics, one fundamental and three antifundamentals.

In the following we show the matching of the elliptic genera along the same lines of the discussion performed in the other examples. In this case we found more convenient[11] to flip the meson M and the baryons $B_{1,2}$, $A_{1,2}^n Q$ and $A_{1,2}^n A_{2,1} \tilde{Q}_{2,1}$ in the dual LG and we will show that the expected J-term in (3.62) can be reconstructed by our procedure using the J-term expected from the duality of subsection 3.2.

At the level of the elliptic genus the identity associated to this duality is

$$
\begin{aligned}
I^{(1,3,\cdot,2,\cdot)}_{\mathrm{SU}(2n+1)}(u;\vec{v};\cdot;\vec{t};\cdot) &= \frac{\prod_{j=0}^{2n-4} \theta(q/(t_1^{j+3} t_2^{2n-j-1} u v_1 v_2 v_3))}{\prod_a \theta(u v_a) \prod_{a<b}(\theta(t_{1,2} v_a v_b)) \prod_{j=0}^{n-3} \theta(t_1^{n-1-j} t_2^{j+2} u v_a v_b)} \\
&\times \frac{1}{\prod_{j=0}^{n-4} \theta(t_1^{n-j-1} t_2^{j+3} v_1 v_2 v_3) \prod_{j=0}^{n} \theta(t_1^{n-j} t_2^{j} u) \prod_{j=0}^{n-1} \prod_a \theta(t_1^{n-j} t_2^{j+1} v_a)}\,. \qquad (3.64)
\end{aligned}
$$

In order to prove this relation we deconfine the two antisymmetric as in Figure 10, obtaining two $\mathrm{USp}(2n)$ gauge nodes, with two bifundamentals $P_{1,2}$.

[11] There are other possible patterns and flippers that can be used to prove the duality through tensor deconfinement, but we found that the choice made here allows for a simpler derivation.

Actually the procedure adopted here is slighlty different from the cases studied above and it deserves some further comment. In the various cases studied so far the two symplectic gauge nodes could be reconfined simultaneously to give the original model. Here such reconfinement must be done in two steps. Indeed there is a field, corresponding to the Fermi ψ in the second quiver of Figure 10 that gives a non-vanishing J-term and which requires, after reconfining one of the two $\mathrm{USp}(2n)$ gauge nodes, to integrate out some massive combinations of chirals and Fermi.

The J-term for the fields ψ is

$$J_\psi = P_1 R_1 + P_2 R_2\,. \tag{3.65}$$

In addition in this phase there are other J-terms, originating from the flippers for M and $B_{1,2}$ discussed above

$$J_{\Psi_M} = \tilde{Q}(P_1 R_1 - P_2 R_2), \quad J_{\Psi_{B_{1,2}}} = P_{1,2}^2 \tilde{Q}^2. \tag{3.66}$$

The $\mathrm{SU}(2n+1)$ gauge node has now $4n$ fundamentals, three antifundamentals and one Fermi. It can be dualized according to the rules explained in subsection 2.3, but in this case with $y = 1$. The dual phase corresponds to the third quiver in Figure 10. In this case the new J-term generated by the duality is $J_\lambda = N_2 \tilde{P}_2 + N_1 \tilde{P}_1$. In addition the other J-terms in this phase are

$$J_{\Psi_{B_1}} = N_1^2, \quad J_{\Psi_{B_2}} = N_2^2, \quad J_{\Psi_M} = q(N_1 \tilde{P}_1 - N_2 \tilde{P}_2) \tag{3.67}$$

where the last term is obtained after integrating out the massive fields.

The last step consists of dualizing the two $\mathrm{USp}(2n)$ groups, each one with $2n+2$ chiral fundamentals, to an LG. The final quiver is the fourth one in Figure 10. In this case there are two J-terms associated to two Fermi singlets $\Psi_{1,2}^{(0)}$. They read $J_{\Psi_{1,2}^{(0)}} = \tilde{A}_{1,2}^{n-2} \tilde{q}^3$. There is a further J term for the field Ψ_M that reads $J_{\Psi_M} = q\tilde{q}$.

Again we have, up to an overall charge conjugation, a model already described in 3.2. We can the use the duality of such model with an LG in order to prove the validity of (3.64) and to reconstruct (up to flippers) the expected J-term (3.62). The details of the derivation are straightforward and we leave them to the interested reader.

3.9 $\mathrm{SU}(2n)$ with four antifundamentals

In this case the dual LG model has four chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

$$
\begin{array}{c|c|c|c|c|c|c}
 & \mathrm{SU}(2n) & \mathrm{SU}(2) & \mathrm{SU}(4) & \mathrm{U}(1)_A & \mathrm{U}(1)_{\tilde{Q}} & \mathrm{U}(1)_{R_0} \\
\hline
A & \begin{smallmatrix}\square\\\square\end{smallmatrix} & \square & \cdot & 1 & 0 & 0 \\
\tilde{Q} & \square & \cdot & \square & 0 & 1 & 0 \\
\hline
B & \cdot & \square & \begin{smallmatrix}\square\\\square\end{smallmatrix} & 1 & 2 & 0 \\
T_n & \cdot & \otimes^{n}_{\mathrm{sym}}\square & \cdot & n & 0 & 0 \\
T_{n-1} & \cdot & \otimes^{n-3}_{\mathrm{sym}}\square & \begin{smallmatrix}\square\\\square\end{smallmatrix} & n+1 & 2 & 0 \\
T_{n-2} & \cdot & \otimes^{n-6}_{\mathrm{sym}}\square & \cdot & n+2 & 4 & 0 \\
\Psi & \cdot & \otimes^{2n-6}_{\mathrm{sym}}\square & \cdot & -2n-2 & -4 & 2
\end{array}
\tag{3.68}
$$

The chirals T_j and B correspond to the following gauge invariant combinations of the charged fields

$$T_n = A^n\,, \qquad T_{n-1} = A^{n-1}(A\tilde{Q})^2\,, \quad T_{n-2} = A^{n-2}(A\tilde{Q})^4\,, \quad B = A\tilde{Q}^2\,, \tag{3.69}$$

while the J-term compatible with the global symmetry is

$$J_\Psi = T_n T_{n-2} + T_{n-1}^2 + B^2 T_n^2 + B T_n T_{n-1}\,. \tag{3.70}$$

We computed and matched the 't Hooft anomalies in the two dual theories. Explicitly we have

$$
\begin{aligned}
&\kappa_{\mathrm{SU}(2)^2} = \tfrac{n(2n-1)}{2}\,, && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n-1)\,,\\
&\kappa_{\mathrm{SU}(4)^2} = n\,, && \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R_0} = 8n\,,\\
&\kappa_{R_0R_0} = 6n+1\,, && \kappa_{A\tilde{Q}} = 0\,.
\end{aligned}
\tag{3.71}
$$

In the following we show the matching of the elliptic genera along the same lines of the discussion performed in the other examples. In this case we will flip the baryons B and $\mathrm{Pf}A_{1,2}$ in the dual LG and we show that the expected J-term in (3.70) can be reconstructed by our procedure using the J-term expected from the duality of subsection 3.2.

At the level of the elliptic genus the identity associated to this duality is

$$
\begin{aligned}
I^{(\cdot,4,\cdot,2,\cdot)}_{\mathrm{SU}(2n)}(\cdot;\vec{v};\cdot;\vec{t};\cdot) \;&= \frac{\prod_{j=0}^{2n-6}\theta(q/(t_1^{j+4}t_2^{2n-j-2}\prod_{a=1}^4 v_a))}{\prod_{a<b}(\theta(t_{1,2}v_av_b)\prod_{j=0}^{n-3}\theta(t_1^{n-1-j}t_2^{j+2}v_av_b))}\\
&\times\frac{1}{\prod_{j=0}^{n-6}\theta(t_1^{n-j-2}t_2^{j+4}\prod_{a=1}^4 v_a)\prod_{j=0}^{n}\theta(t_1^{n-j}t_2^{j})}\,.
\end{aligned}
\tag{3.72}
$$

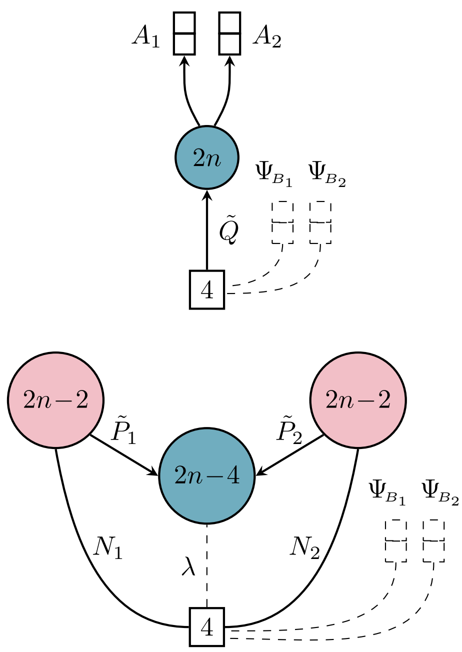

Figure 11: Deconfinement steps of SU($2n$) with two antisymmetrics and four antifundamentals.

In order to prove this relation we deconfine the two antisymmetric as in Figure 11, obtaining two USp($2n-2$) gauge nodes, with two bifundamentals $P_{1,2}$. The original antisymmetric chirals $A_{1,2}$ correspond to the combinations $P_{1,2}^2$. The J-term at this stage is the one flipping the operator B, corresponding to

$$J_{\Psi_{B_{1,2}}} = \tilde{Q}^2 P_{1,2}^2 \,. \tag{3.73}$$

The SU($2n$) gauge theory has $4n-4$ fundamentals and four antifundamentals. It can be dualized according to the rules explained in subsection 2.3, and it gives origin to the third quiver in Figure 11.

In this case the new J-term generated by the duality is $J_\lambda = N_2\tilde{P}_2 + N_1\tilde{P}_1$. In addition the J-terms for Ψ_{B_1} and Ψ_{B_2} become $J_{\Psi_{B_1}} = N_1^2$ and $J_{\Psi_{B_2}} = N_2^2$ (where the flavor indices are left implicit in these relations).

The last step consists of dualizing the two USp($2n-2$) groups, each one with $2n$ chiral fundamentals, to an LG. The final quiver is the fourth one in Figure 9. In this case there are two J-terms associated to two Fermi singlets $\Psi_{1,2}^{(0)}$. They read $J_{\Psi_{1,2}^{(0)}} = \tilde{A}_{1,2}^{n-4}\tilde{q}^4$.

Again we have, up to an overall charge conjugation, a model already described in 3.2. We can the use the duality of such model with an LG in order to prove the validity of (3.72) and to reconstruct (up to flippers) the expected J-term (3.70). The details of the derivation are straightforward and we leave them to the interested reader.

3.10 $\mathrm{SU}(2n+1)$ with four antifundamentals

Here we conclude this section with the last case of our classification of 2d $\mathcal{N} = (0,2)$ gauge/LG dualities for $\mathrm{SU}(N)$ gauge theories with two antisymmetric chirals. In this case the dual LG model has three chiral gauge invariant operators corresponding to chiral fields interacting through a J-term with a Fermi multiplet. The field content of the gauge theory and of the dual LG model is represented in the table below.

$$\tag{3.74}$$

	$\mathrm{SU}(2n+1)$	$\mathrm{SU}(2)$	$\mathrm{SU}(4)$	$\mathrm{U}(1)_A$	$\mathrm{U}(1)_Q$	$\mathrm{U}(1)_{R_0}$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	$\cdot$	1	0	0
$\tilde{Q}$	$\square$	$\cdot$	$\square$	0	1	0
B	$\cdot$	$\square$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	2	0
P_{n+1}	$\cdot$	$\otimes^{n-1}_{\mathrm{sym}}\square$	$\square$	$n+1$	1	0
P_{n+2}	$\cdot$	$\otimes^{n-4}_{\mathrm{sym}}\square$	$\overline{\square}$	$n+2$	3	0
Ψ	$\cdot$	$\otimes^{2n-5}_{\mathrm{sym}}\square$	$\cdot$	$-2n-3$	-4	2

The chirals P_j and B correspond to the following gauge invariant combinations of the charged fields

$$P_{n+1} = A^n(A\tilde{Q})\,, \qquad P_{n+2} = A^{n-1}(A\tilde{Q})^3\,, \quad B = \tilde{A}Q^2\,, \tag{3.75}$$

while the most general J-term compatible with the global symmetry is

$$J_\Psi = P_{n+1}P_{n+2} + BP_{n+1}^2\,. \tag{3.76}$$

We have computed the 't Hooft anomalies in the two phases and they match. Explicitly we have

$$\begin{aligned}
&\kappa_{\mathrm{SU}(2)^2} = \tfrac{n(2n+1)}{2}\,, && \kappa_{AA} = -\kappa_{AR_0} = 2n(2n+1)\,,\\
&\kappa_{\mathrm{SU}(4)^2} = n\,, && \kappa_{\tilde{Q}\tilde{Q}} = -\kappa_{\tilde{Q}R_0} = 8n\,,\\
&\kappa_{R_0R_0} = 6n+4\,, && \kappa_{A\tilde{Q}} = 0\,.
\end{aligned} \tag{3.77}$$

In the following we show the matching of the elliptic genera along the same lines of the discussion performed in the other examples.

Actually in this case we need to adopt the same strategy used in the cases analyzed in subsections 3.1 and 3.2 in order to trade the two antisymmetric with two symplectic gauge groups. Concretely, we flip them by using the confining duality reviewed in subsection 2.1.

In this case we also flip the baryon B in the dual LG and we show that the expected J-term in (3.76) can be reconstructed by our procedure using the J-term expected from the duality of subsection 3.1.

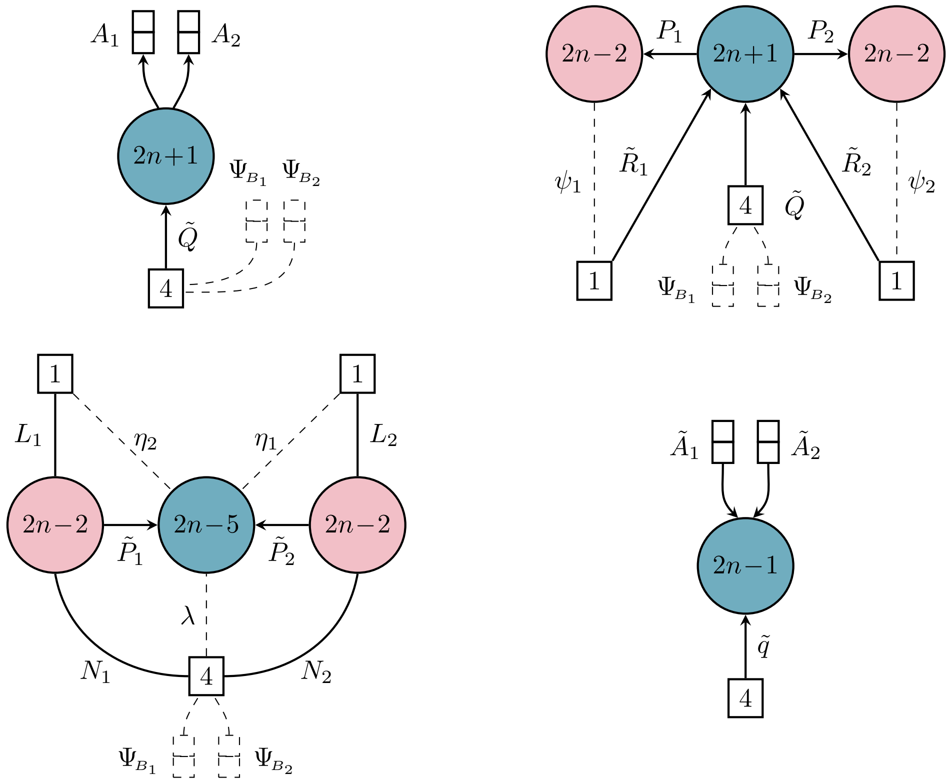

Figure 12: Deconfinement steps of $\mathrm{SU}(2n+1)$ with two antisymmetrics and four antifundamentals.

At the level of the elliptic genus the identity associated to this duality is

$$I^{(\cdot,4,\cdot,2,\cdot)}_{\mathrm{SU}(2n+1)}(\cdot;\vec{v};\cdot;\vec{t};\cdot) = \frac{\prod_{j=0}^{2n-5}\theta(q/(t_1^{j+4}t_2^{2n-j-1}\prod_{a=1}^4 v_a))}{\prod_{a<b}\theta(t_{1,2}v_a v_b)}$$
$$\times\frac{1}{\prod_{a<b<c}\prod_{j=0}^{n-4}\theta(t_1^{n-j-1}t_2^{j+3}v_a v_b v_c)\prod_a\prod_{j=0}^{n-1}\theta(t_1^{n-j}t_2^{j+1}v_a)}\,. \tag{3.78}$$

In order to prove this relation we deconfine the two antisymmetrics as in Figure 12, obtaining two $\mathrm{USp}(2n-2)$ gauge nodes, with two bifundamentals $P_{1,2}$ and two fundamental Fermi $\psi_{1,2}$. The original antisymmetric chirals $A_{1,2}$ correspond to the combinations $P_{1,2}^2$. There are also two J-terms $J_{\psi_{1,2}} = P_{1,2}\tilde{R}_{1,2}$ and in addition there are J-terms flipping the operators $B_{1,2}$, corresponding to

$$J_{\Psi_{B_{1,2}}} = \tilde{Q}^2 P_{1,2}^2\,. \tag{3.79}$$

As discussed in subsection 3.1 in this phase we must impose a further constraint on the charges in order to prevent the generation of an axial symmetry for each $\mathrm{USp}(2n-2)$

gauge symmetry. We refer the reader to the discussion there for further comments on this problem.

The $\mathrm{SU}(2n+1)$ gauge theory has $4n-4$ fundamentals and four antifundamentals. It can be dualized according to the rules explained in subsection 2.3, and it gives origin to the third quiver in Figure 12.

In this case the new J-terms generated by the duality are $J_\lambda = N_2\tilde{P}_2 + N_1\tilde{P}_1$ and $J_{\eta_{1,2}} = L_{1,2}\tilde{P}_{1,2}$. In addition the J-terms for Ψ_{B_1} and Ψ_{B_2} become $J_{\Psi_{B_1}} = N_1^2$ and $J_{\Psi_{B_2}} = N_2^2$ respectively (again omitting the antisymmetric flavor indices).

The last step consists of dualizing the two $\mathrm{USp}(2n-2)$ groups, each one with $2n$ chiral fundamentals, to an LG. The final quiver is the fourth one in Figure 10. In this case there are two J-terms associated to two Fermi singlets $\Psi_{1,2}^{(0)}$. They read $J_{\Psi_{1,2}^{(0)}} = C_{1,2}\tilde{A}_{1,2}^{n-4}\tilde{q}^3$.

Again we have, up to an overall charge conjugation, a model already described in 3.2. We can the use the duality of such model with an LG in order to prove the validity of (3.78) and to reconstruct (up to flippers) the expected J-term (3.76). The details of the derivation are straightforward and we leave them to the interested reader.

4 A 4d parent for $\mathrm{SU}(2n)$ with $(A, \tilde{A})$ and $4\,\square$

In this section we discuss a 4d duality that, once topologically twisted on a S^2 using the prescription of [1], gives origin to a 2d gauge/LG duality. The 2d model corresponds to a 2d duality studied in [9] using tensor deconfinement. The claim made in [9] was that the model did not have a 4d origin in terms of any known 4d s-confining duality in the classification of [26]. Such a classification is complete for models with vanishing superpotential.

Despite this fact, here we obtain a parent 4d duality between two gauge theories in which the dual phase reduces in 2d to the expected LG model.

Here, in order to prove the 4d duality we will use tensor deconfinement. Furthermore, the presence of the superpotential will trigger non-trivial RG flows in the dual phases. The 4d original model corresponds to $\mathrm{SU}(2n)$ SQCD with four fundamental flavors and one antisymmetric flavor. We turn on the following superpotential

$$W = A^{n-1}Q_3Q_4 + \sum_{i=1}^{n-1}\beta_i \mathrm{Tr}(A\tilde{A})^i + s_A\,\mathrm{Pf}A + s_{\tilde{A}}\,\mathrm{Pf}\tilde{A} + M_aQ_a\tilde{Q} + \gamma\tilde{A}^{n-2}\tilde{Q}^4\,. \quad (4.1)$$

with $a = 1, 2$. We then deconfine the antisymmetric A obtaining a new $\mathrm{USp}(2n-2)$ gauge group with an SU/USp bifundamental P. In addition we consider two $\mathrm{USp}(2n-2)$ fundamentals $R_{3,4}$ as in Figure 13. The original fields A and $Q_{3,4}$ correspond in

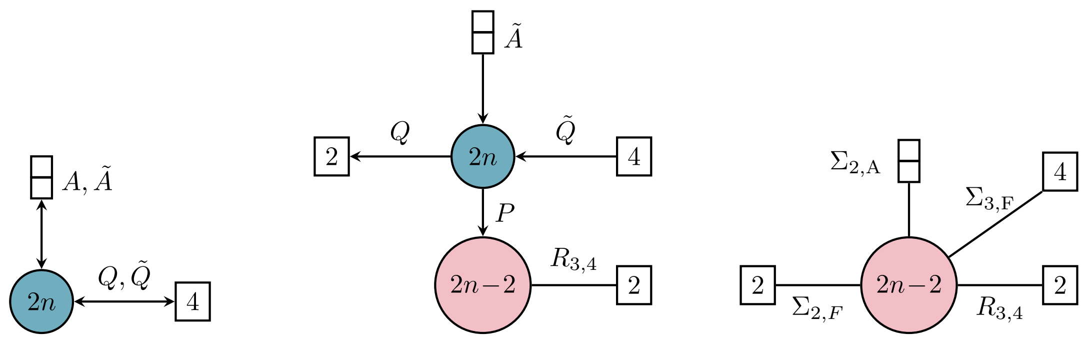

Figure 13: In this Figure we provide a quiver description of the 4d confining gauge theory studied in this section. The original model corresponds to SU($2n$) with four fundamental flavors and one antisymmetric flavor. The theory has a non-trivial superpotential given in formula (4.1). The second quiver corresponds to the model where we have deconfined the SU($2n$) antisymmetric tensor. The third model is obtained after using the s-confining duality of [26] on the SU($2n$) gauge node.

this phase to the combinations P^2 and $PR_{3,4}$ respectively. The superpotential for this phase is

$$W = \sum_{i=1}^{n-1} \beta_i \mathrm{Tr}(P^2 \tilde{A})^i + s_{\tilde{A}} \,\mathrm{Pf} \tilde{A} + M_a Q_a \tilde{Q} + \gamma \tilde{A}^{n-2} \tilde{Q}^4 \,. \tag{4.2}$$

To proceed we notice that the SU($2n$) gauge node has one conjugate antisymmetric, $2n$ fundamentals and 4 antifundamentals. The model is then s-confining in terms of the following SU($2n$) gauge invariant combinations

$$\begin{array}{lllll} \Sigma_1 = \tilde{A}^{n-1}\tilde{Q}^2, & \Sigma_{2,A} = \tilde{A}P^2, & \Sigma_{2,F} = \tilde{A}PQ, & \Sigma_{2,s} = \tilde{A}Q^2 & \\ \Sigma_{3,F} = \tilde{Q}P, & \Sigma_{3,s} = \tilde{Q}Q, & \Sigma_4 = Q^2 P^{2n-2}, & \Sigma_5 = \mathrm{Pf}\, \tilde{A}, & \Sigma_6 = \tilde{A}^{n-2}\tilde{Q}^4 \end{array} \tag{4.3}$$

where $\Sigma_{2,A}$ is in the (traceless) antisymmetric representation of the leftover USp($2n-2$) gauge group, while $\Sigma_{3,F}$ is in the fundamental of such USp($2n-2$). The fields $\Sigma_{5,6}$ and $\Sigma_{3,s}$ are set to zero in the chiral ring by the superpotential (4.2) and we can ignore them in the following. The other Σ-fields are USp($2n-2$) singlets. The superpotential for this phase, after integrating out the massive fields, is

$$W = \Sigma_1 \Sigma_{3,F}^2 (\Sigma_{2,s} \Sigma_{2,A}^{n-2} + \Sigma_{2,A}^{n-3} \Sigma_{2,F}^2) + \sum_{i=2}^{n-1} \beta_i \,\mathrm{Tr}(\Sigma_{2,A})^i + \Sigma_4 \Sigma_1^2 \,. \tag{4.4}$$

In this way we have obtained a duality between SU($2n$) with an antisymmetric flavor, four fundamental flavors and superpotential (4.1), and an USp($2n-2$) gauge theory with eight fundamental, one antisymmetric and superpotential (4.4). This last theory is not confining but it has 72 many (self [12]–) dual phases (with a different structure of flippers. Indeed the global symmetry is known to enhance to $E_7 \times \mathrm{U}(1)$ [33]).

This duality gives origin to the 2d gauge/LG duality discussed in [9] once we compactify the models using the prescription of [1] by fixing the R charges of the fields $R_{\tilde{Q},\tilde{A},A} = 0$ and $R_Q = 1$. The former become chirals in 2d $\mathcal{N} = (0,2)$ while the latter do not give rise to any 2d field. Then, the singlets β_i, s_A, $\tilde{s}_{\tilde{A}}$ and γ have R charge $R = 2$ and become Fermi fields in 2d, that we denote as Ψ_{β_i}, Ψ_{s_A},$\Psi_{s_{\tilde{A}}}$ and Ψ_γ respectively. The singlets M_a, on the other hand, have R charge 1 and they do not give rise to any 2d field.

The 4d superpotential (4.1) of the electric theory gives origin to the 2d J-terms

$$J_{\psi_{\beta_i}} = \mathrm{Tr}(A\tilde{A})^i, \quad J_{\psi_{s_A}} = \mathrm{Pf}A, \quad J_{\psi_{s_{\tilde{A}}}} = \mathrm{Pf}\tilde{A}, \quad J_{\psi_\gamma} = \tilde{A}^{n-2}\tilde{Q}^4, \tag{4.5}$$

with $i = 2, \ldots, n-1$, where we kept the same names for the 2d $\mathcal{N} = (0,2)$ chirals of the corresponding 4d $\mathcal{N} = 1$ chiral multiplets.

By following the duality map, in the dual $\mathrm{USp}(2n-2)$ gauge theory the antisymmetric $\Sigma_{2,A}$, the fundamentals $\Sigma_{3,F}$ and the singlet Σ_1 have R charge $R = 0$ and they survive as 2d $\mathcal{N} = (0,2)$ chirals while the fields $\Sigma_{2,F}$ and $R_{3,4}$ have R charge $R = 1$ and do not survive in 2d.

On the other hand, the singlets $\Sigma_{2,s}$ and Σ_4 have R charge $R = 2$ and they give origin to the Fermi fields $\psi_{\Sigma_{2,s}}$ and ψ_{Σ_4} respectively. The 4d superpotential (4.4) of the dual theory gives origin to the 2d J-terms

$$J_{\psi_{\beta_i}} = \mathrm{Tr}\left(\Sigma_{2,A}\right)^i, \quad J_{\psi_{\Sigma_{2,s}}} = \Sigma_1 \Sigma_{3,F}^2 \Sigma_{2,A}^{n-2}, \quad J_{\psi_{\Sigma_4}} = \Sigma_1^2\,, \tag{4.6}$$

with $i = 2, \ldots, n-2$, where again we kept the same names for the 2d $\mathcal{N} = (0,2)$ chirals of the corresponding 4d $\mathcal{N} = 1$ chiral multiplets.

The J-terms in (4.5) and (4.6) are equivalent to the 2d superpotentials discussed in formulas (4.72) and (4.76) of [9], where the dual phase was further dualized to an LG.

Summarizing, in this section we have obtained a parent 4d duality between two gauge theories that, once compactified to 2d, gives origin to the 2d gauge/LG duality studied in [9]. The 4d duality in this case does not correspond to an s-confining theory, even if it is still possible that there exist other 4d s-confining models that can be reduced to the same 2d dualities. We leave such possibility to future investigations.

[12]In the sense that the dual theories have an $\mathrm{USp}(2n-2)$ gauge group with an antisymmetric and eight fundamentals but a different structure of singlets and superpotential.

5 Dualities involving antisymmetric Fermi fields

In this section we focus on $\mathrm{SU}(n)$ and $\mathrm{USp}(2n)$ gauge theories with (anti)-fundamental chirals and with a Fermi field in the antisymmetric representation. The situation is different from the cases treated above and in [7–9], because in these cases the technique of tensor deconfinement does not apply. This is due to the fact that, to the best of our knowledge, the fundamental gauge/LG dualities proposed in the literature never include antisymmetric Fermi fields charged under a non-abelian flavor symmetry. This constitutes an obstruction to proving dualities using the deconfining technique. Despite this fact, here we observe that some dualities with charged antisymmetric Fermi fields can be obtained by using the prescription of [1] from 4d parent dualities proposed in the literature starting from the matching of the superconformal index. We further check the 2d dualities obtained by matching the 't Hooft anomalies.

5.1 A gauge/LG duality with an antisymmetric Fermi

The first 4d duality under investigation has been proposed in [30, 34] and further investigated in [35], where it was referred to as SWV duality. A proof of the dualities in terms of other fundamental dualities was then provided in [20].

The electric theory in this duality is an $\mathrm{SU}(n+1)$ gauge theory with an antisymmetric A, $2n$ antifundamentals $\tilde{Q}$ and $n+3$ fundamentals Q with 4d superpotential

$$W^{\text{ele}}_{\text{SWV}} = A\tilde{Q}^2\,. \tag{5.1}$$

The dual theory is a WZ model, described by the electric baryon $B = Q^{n+1}$ and meson $M = Q\tilde{Q}$ with superpotential

$$W^{\text{mag}}_{\text{SWV}} = BM^2\,. \tag{5.2}$$

We are interested in a flipped version of this duality, where we break the global $\mathrm{SU}(n+3)$ symmetry by partially flipping the baryonic operator B. On the electric side, we consider the split $Q \to (Q_1, Q_2)$, corresponding to the decomposition of $\mathrm{SU}(n+3) \to \mathrm{SU}(n+1) \times \mathrm{SU}(2)$. Such symmetry breaking pattern is enforced in the electric superpotential by adding a singlet b and considering the flipped superpotential

$$W^{\text{ele}}_{\text{SWV}} = A\tilde{Q}^2 + bQ_1^{n+1}\,. \tag{5.3}$$

On the magnetic side, under the symmetry breaking pattern above, the baryon B is decomposed into three fields: an $\mathrm{SU}(n+1)$ antisymmetric B_A, an $\mathrm{SU}(n+1) \times \mathrm{SU}(2)$ bifundamental B_V and a singlet B_S. Similarly, the meson M splits into M_1 and M_2

respectively in the fundamental representation of $\mathrm{SU}(n+1)$ and $\mathrm{SU}(2)$. The dual superpotential becomes

$$W_{\mathrm{SWV}}^{\mathrm{mag}} = B_A M_1^2 + B_V M_1 M_2 + B_S M_2^2 + b B_S \,, \tag{5.4}$$

and after integrating out the massive fields it reduces to

$$W_{\mathrm{SWV}}^{\mathrm{mag}} = B_A M_1^2 + B_V M_1 M_2 \,, \tag{5.5}$$

as expected from the addition of the baryonic flipper in (5.3).

The superconformal index still matches across the dual phases, as the addition of the flipper just moves the relative elliptic gamma function from one side of the duality to the other without modifying the balancing condition, or equivalently, the requirements from the anomaly freedom.

In the next step we fix the non-negative integer R charges of the flipped version of the duality. We fix the R charge of the antisymmetric to $R_A = 2$ and, consistently with the anomaly freedom condition, we fix the R charges of the fields Q_2 to $R_{Q_2} = 1$. The charges of Q_1 and $\tilde{Q}$ are then vanishing, consistently with the global symmetries. The singlet b in (5.3) on the other hand has $R_b = 2$.

The 2d theory obtained using such assignation of charges is an $\mathrm{SU}(n+1)$ electric gauge theory with an antisymmetric Fermi Ψ_A, $2n$ antifundamental chirals $\tilde{Q}$ and $n+1$ fundamental chirals Q with 2d J-terms

$$J_{\Psi_A} = \tilde{Q}^2 \,, \qquad J_{\Psi_B} = Q^{n+1} \,, \tag{5.6}$$

where the Fermi ψ_b descend from the 4d singlet b.

In the 4d dual theory the R-charges that can be read from the duality map are $R_{B_A} = 2$, $R_{M_1} = 0$ and $R_{M_2} = R_{B_V} = 1$. The field B_A becomes a Fermi field, denoted as ψ, in the antisymmetric representation of the $\mathrm{SU}(n+1)$ flavor symmetry. The meson M_1 becomes a chiral, denoted by ϕ_M, in the antifundamental representation of the $\mathrm{SU}(n+1)$ flavor symmetry (and in the fundamental of the other $\mathrm{USp}(2n)$ flavor symmetry). On the other hand, the fields B_S and M_2 do not give rise to any 2d field.

The magnetic description is an LG model with J-term $J_\Psi = \Phi_M^2$ where we can further associate Φ_M to the combination $Q\tilde{Q}$. This relation and the structure of the J-term fix the charges of the fields in the dual picture. This results into the table of global charges below.

	$\mathrm{USp}(2n)$	$\mathrm{SU}(n+1)$	$\mathrm{U}(1)_A$	$\mathrm{U}(1)_Q$	$\mathrm{U}(1)_R$
Ψ_A	$\cdot$	$\cdot$	1	0	1
$\tilde{Q}$	$\square$	$\cdot$	$-\frac{1}{2}$	0	0
Q	$\cdot$	$\square$	0	1	0
Ψ_B	$\cdot$	$\cdot$	0	$-n-1$	1
Φ_M	$\square$	$\overline{\square}$	1	$-\frac{1}{2}$	0
Ψ	$\cdot$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	-2	1	1

(5.7)

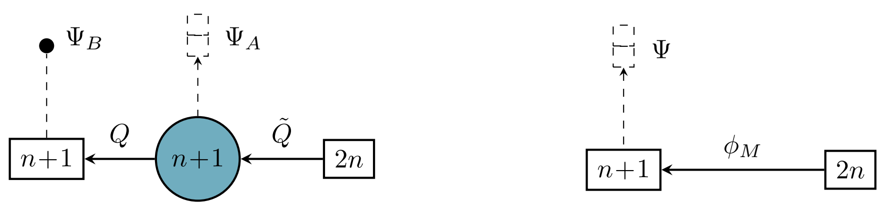

Figure 14: In this figure we represent the quivers associated to the duality between $\mathrm{SU}(n{+}1)$ with an antisymmetric Fermi and the LG model. Dashed lines are for Fermi fields while continuous lines are for chiral multiplets. The dots refer to the singlets.

We now provide some checks of the duality proposed here obtained from the reduction of the parent 4d SWV duality. First of all, we have computed the 't Hooft anomalies and checked their matching. Explicitly, we have

$$\begin{aligned} &\kappa_{\mathrm{SU}(n+1)^2} = \frac{n+1}{2}\,, && \kappa_{\mathrm{USp}(2n)^2} = n+1\,, && \kappa_{QQ} = \kappa_{AA} = \kappa_{QA} = 0\,, \\ &\kappa_{RR} = \frac{3n(n+1)}{2}\,, && \kappa_{AR} = \frac{n(n+1)}{2}\,, && \kappa_{RQ} = -n(n+1)\,. \end{aligned} \tag{5.8}$$

The validity of the duality also implies the relation between the ellitpic genera

$$\begin{aligned} I_{\text{ele}} = {} & \frac{(q;q)_\infty^{2n+2}}{(n+1)!} \oint_{\text{JK}} \prod_{i=1}^{n+1} \frac{\mathrm{d}z_i}{2\pi \mathrm{i} z_i} \frac{\prod_{1\le i<j\le n+1} \theta(qTz_iz_j)\theta((z_i/z_j)^{\pm1})}{\prod_{i=1}^{n+1}\left(\prod_{m=1}^{n+1}\theta(z_is_m)\prod_{k=1}^{n}\theta(t_k^{\pm1}z_i^{-1}/\sqrt{T})\right)} \\ & \times \theta\left(q\prod_{\ell=1}^{n+1}s_\ell^{-1}\right) = \frac{\prod_{1\le m<\ell\le n+1}\theta(qTs_m^{-1}s_\ell^{-1})}{\prod_{m=1}^{n+1}\prod_{k=1}^{n}\theta(s_mt_k^{\pm1}/\sqrt{T})} = I_{\text{mag}}\,, \end{aligned} \tag{5.9}$$

with $\prod_{i=1}^{n+1} z_i = 1$. We explitly checked the validity of this identity for the cases $n = 1, 2$, where the duality reduces to a flipped version of the ones studied in subsection 2.3. Indeed, in the first case the antisymmetric Fermi disappears, while in the second case it is equivalent to an antifundamental Fermi field.

A further check can be obtained through a non-anomalous gauging of a subgroup of the USp($2n$) flavor symmetry group. For example, let us start by considering the case of $n = 2m - 1$ such that the flavor symmetry is USp($4m - 2$). Then, we gauge an USp($2m - 2$) subgroup of this symmetry. The final USp($2m$) $\times$ USp($2m - 2$) is then non-anomalous and the superpotential for this model becomes

$$W = \Psi_A \tilde{Q}_1^2 + \Psi_A \tilde{Q}_2^2 + \Psi_B Q^{2m} , \tag{5.10}$$

where Q_1 is the SU($2m$) $\times$ USp($2m - 2$) bifundamentals while Q_2 represents the other $2m$ SU($2m$) fundamentals whose flavor symmetry is not gauged.

The USp($2m - 2$) gauge theory has $2m$ fundamentals and it can be dualized into an LG model. The superpotential after this duality becomes

$$W = \Psi_A A + \Psi_A \tilde{Q}_2^2 + \psi_\sigma \mathrm{Pf} A + \Psi_B Q^{2m} . \tag{5.11}$$

The Fermi Ψ_A and the chiral antisymmetric $A = Q_1^2$ are massive and can be integrated out, leaving us with

$$W = \Psi_A A + \Psi_A \tilde{Q}_2^2 + \psi_\sigma \mathrm{Pf} A + \Psi_B Q^{2m} . \tag{5.12}$$

By integrating out the massive fields the superpotential for the remaining SU($2m$) gauge theory becomes

$$W = \psi_\sigma \tilde{Q}_2^{2m} + \Psi_B Q^{2m} , \tag{5.13}$$

that is dual to an LG with a meson $\Phi_M = Q\tilde{Q}_2$ with superpotential

$$W = \hat{\Psi} \det \Phi_M . \tag{5.14}$$

The same gauging can be performed in the dual phase in which the model becomes

$$W = \Psi(\Phi_{M_1}^2 + \Phi_{M_2}^2) , \tag{5.15}$$

where Φ_{M_1} is charged under the USp($2m - 2$) gauge symmetry, while Φ_{M_2} is charged under the USp($2m$) flavor symmetry. The USp($2m - 2$) gauge group is dual to an LG model and we get

$$W = \Psi(\mathcal{A} + \Phi_{M_2}^2) + \Psi_0 \, \mathrm{Pf} \mathcal{A} , \tag{5.16}$$

with $\mathcal{A} = \Phi_{M_1}^2$. By integrating out the massive fields, such superpotential becomes

$$W = \Psi_0 \, \mathrm{Pf}(\Phi_{M_2})^2 = \Psi_0 \det \Phi_{M_2} , \tag{5.17}$$

which is equivalent to (5.14).

5.2 A gauge/gauge duality with an antisymmetric Fermi

The second duality in presence of charged antisymmetric Fermi fields that we consider relates two symplectic gauge theories with USp($2n$) and USp($2n-2$) gauge group respectively. Again, the duality is obtained from a 4d parent, that was originally proposed in [30] from the matching of the associated superconformal indices obtained in [36]. The duality was then derived from a physical perspective in [21].

The 4d duality relates models characterized by symplectic gauge groups, an antisymmetric field together with fundamental flavors which interact with the antisymmetric, breaking the flavor symmetry with various possible patterns. Here we focus on a specific pattern, suitable to give origin to a 2d $\mathcal{N}=(0,2)$ duality with a charged antisymmetric Fermi field. In this case the flavor symmetry is $\mathrm{SU}(4)\times\mathrm{USp}(4n-2)$. There are four fundamentals P and $4n-2$ fundamentals Q, which interact with the USp($2n$) antisymmetric A through a superpotential $W=AQ^2$. The dual picture corresponds to an USp($2n-2$) gauge theory with four fundamentals p, $4n-2$ fundamentals q and an antisymmetric, denoted as a. The superpotential of this dual phase is

$$W = NM^2 + Mpq + aq^2, \tag{5.18}$$

where the singlets N and M correspond in the electric phase to the gauge invariant combinations P^2 and PQ.

Again, we consider a flipped version of this duality by breaking the SU(4) flavor symmetry to $\mathrm{SU}(2)_1\times\mathrm{SU}(2)_2$, denoting the two fundamentals as P_1 and P_2 and flipping the combination P_2^2 by adding a singlet B in the electric phase. The electric theory has then superpotential

$$W_{USp(2n)} = AQ^2 + BP_2^2. \tag{5.19}$$

On the magnetic side the superpotential becomes

$$W_{USp(2n-2)} = M_1p_1q + M_2p_2q + aq^2 + M_1^2N_1 + M_1M_2L, \tag{5.20}$$

where $M_1=QP_1$, $M_2=QP_2$, $L=P_1P_2$ and $N_1=P_1^2$.

We then compactify the theory on S^2 by fixing $R_{P_1}=R_Q=0$, $R_A=R_S=2$ and $R_{P_2}=1$. In this way we obtain a 2d $\mathcal{N}=(0,2)$ USp($2n$) electric gauge theory with an antisymmetric Fermi Ψ_A, $4n-2$ fundamental chirals Q and 2 fundamental chirals P_1 with J-terms $Y_{\Psi_A}=Q^2$ and $J_{\Psi_B}=P_1^2$. In the dual theory the R charges, read from the duality map, are $R_q=R_{M_1}=0$, $R_a=R_{N_1}=R_{p_1}=2$ and $R_{M_2}=R_L=R_{p_2}=1$. The magnetic description is an USp($2n-2$) gauge theory with an antisymmetric Fermi Ψ_a, $4n-2$ fundamental chirals q and 4 fundamentals Fermi multiplets Ψ_{p_1}, with J-terms $J_{\Psi_{p_1}}=q\Phi_{M_1}$, $J_{\Psi_a}=q^2$ and $J_{\Psi_{N_1}}=\Phi_{M_1}^2$.

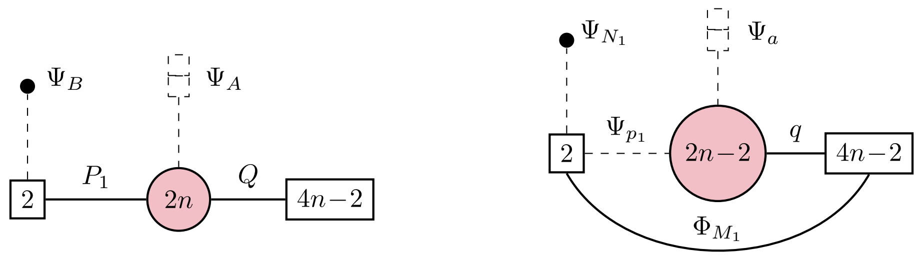

Figure 15: In this figure we represent the quivers associated to the duality between USp($2n$) and USp($2n-2$) with an antisymmetric Fermi. Dashed lines are for Fermi fields while continuous lines are for chiral multiplets. The dots refer to the singlets.

The duality map in 2d can be read once we associate the chiral Φ_{M_1} to the electric mesonic combination P_1Q. The global charges for this duality are then summarized in the table below.

	USp($2n$)	USp($4n-2$)	SU(2)	U(1)$_A$	U(1)$_P$	U(1)$_R$
Ψ_A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\cdot$	$\cdot$	1	0	1
Q	$\square$	$\square$	$\cdot$	$-\frac{1}{2}$	0	0
P_1	$\square$	$\cdot$	$\square$	0	1	0
Ψ_B	$\cdot$	$\cdot$	$\cdot$	0	-2	1
	USp($2n-2$)	USp($4n-2$)	SU(2)	U(1)$_A$	U(1)$_P$	U(1)$_R$
q	$\square$	$\square$	$\cdot$	$-\frac{1}{2}$	0	0
Φ_{M_1}	$\cdot$	$\square$	$\square$	$-\frac{1}{2}$	1	0
Ψ_{p_1}	$\square$	$\cdot$	$\square$	1	1	1
Ψ_a	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\cdot$	$\cdot$	1	0	1
Ψ_{N_1}	$\cdot$	$\cdot$	$\cdot$	1	-2	1

(5.21)

As a check we computed the 't Hooft anomalies for this duality and we have showed that they match. We have

$$\begin{aligned} &\kappa_{PP} = \kappa_{AA} = \kappa_{AP} = 0\,, && \kappa_{RP} = 2(1-2n)\,, && \kappa_{RA} = n(2n-1)\,, \\ &\kappa_{\mathrm{SU}(2)^2} = n\,, && \kappa_{\mathrm{USp}(4n-2)^2} = 2n\,, && \kappa_{RR} = 4n^2-1\,. \end{aligned} \tag{5.22}$$

For completeness we also provide the expression for the elliptic genus in the electric and in the magnetic case. On the electric side we have

$$I_{\mathrm{ele}} = \frac{(q;q)_\infty^{2n}\theta(qT)^{n-1}\theta\left(\frac{q}{t_1t_2}\right)}{n!\,2^n} \oint_{\mathrm{JK}} \prod_{i=1}^{n} \frac{\prod_{i<j}\theta(z_i^{\pm1}z_j^{\pm1})\theta(qTz_i^{\pm1}z_j^{\pm1})\prod_{i=1}^{n}\theta(z_i^{\pm2})}{\prod_{i=1}^{n}\prod_{a=1}^{2}\theta(z_i^{\pm1}t_a)\prod_{\ell=1}^{2n-1}\theta(z_i^{\pm1}T^{-\frac{1}{2}}s_\ell^{\pm1})} \tag{5.23}$$

while on the dual side we have

$$I_{\text{mag}} = \frac{(q;q)_\infty^{2n-2}\theta(qT)^{n-2}\theta\left(\frac{qT}{t_1t_2}\right)}{(n-1)!\,2^{n-1}\prod_{\ell=1}^{2n-1}\theta(s_\ell^{\pm 1}t_{1,2}T^{-\frac{1}{2}})}$$
$$\oint_{\text{JK}}\prod_{i=1}^{n-1}\frac{\prod_{i<j}\theta(z_i^{\pm 1}z_j^{\pm 1})\theta(qTz_i^{\pm 1}z_j^{\pm 1})\prod_{i=1}^{n-1}\left(\theta(z_i^{\pm 2})\prod_{a=1}^{2}\theta\left(\frac{qT}{t_a}z_i^{\pm 1}\right)\right)}{\prod_{i=1}^{n-1}\prod_{\ell=1}^{2n-1}\theta(z_i^{\pm 1}T^{-\frac{1}{2}}s_\ell^{\pm 1})}\,. \quad (5.24)$$

6 Conclusions

In this paper we have studied 2d $\mathcal{N}=(0,2)$ dualities with antisymmetric matter, extending the program started in [9]. The first class of dualities under investigation correspond to SU(N) gauge theories with 2 antisymmetric, n_f fundamental and n_a antifundamental chirals, with $n_f+n_a=4$. Our main claim is that such models have a dual description in terms of LG models, with chiral and Fermi multiplets and non-trivial J-terms. Such gauge/LG dualities cannot in general be predicted from the prescription of [1] by a 4d/2d reduction of s-confining dualities and a check of their validity requires, in principle, a pure 2d analysis. We have checked such claims by verifying the 't Hooft anomaly matching and by studying the elliptic genus. Our analysis has shown that the matching of the elliptic genera follows from other *basic* identities that can be obtained from the dimensional reduction of 4d s-confining dualities. By assuming the matching of the 4d index on $S^2\times T^2$ defined in [2, 37, 38], the matching of the elliptic genera for such *basic* dualities follows from the prescription of [1]. The basic identity employed here have been reviewed in section 2. A crucial relation is the one discussed in subsection 2.1. Despite the fact that the field content in this case admits an additional axial symmetry, the identity 2.4 holds only if this extra symmetry is absent. This suggests that, on the field theory side, some non-perturbative effect must be invoked on the gauge side of the duality to lift such dangerous symmetry. As originally discussed in [1, 7], this expectation is reminiscent of similar results obtained in the circle reduction of 4d dualities, keeping the effects of the KK monopole when considering the 3d effective dualities at finite size of the circle [31]. Here we have used the *basic* identity for this 2d duality under the assumption that such a non-perturbative effect lifts the axial symmetry from the spectrum. As we commented in the introduction one can think of such a 2d duality as arising from a 2d boundary one [29] of the 3d effective theory on the circle. In such a case the matching of the half-index does not depend on the axial symmetry, which is expected to be lifted by the presence of the monopole superpotential in the bulk theory. We refer the reader to [39] for examples of this kind. In this paper we have left to future investigation an appropriate discussion on such non-perturbative effect, and looked at the consequences of the duality of subsection 2.1. We have observed that, by deconfining the antisymmetric chirals using this duality and then dualizing the original $\mathrm{SU}(N)$ gauge node, there is no need to call for non-perturbative effects anymore, and it is then possible to re–confine the antisymmetric chirals using the ordinary gauge/LG for $\mathrm{USp}(2n)$ with $2n+2$ fundamental chirals. By iterating the procedure, we have been able to find the expected identities for the proposed dualities with $n_f = 4$ and $n_a = 0$. Furthermore, we have shown that all the other dualities with $n_a > 0$ arise as a consequence of the ones with $n_a = 0$.

In the second part of the paper we studied a 4d duality between a $\mathrm{SU}(2n)$ theory with an antisymmetric and four fundamental flavors, and a $\mathrm{USp}(2n)$ one with an antisymmetric and eight fundamentals. In absence of a superpotential the former has a nine dimensional Cartan for the non-anomalous non–R global symmetry group, compatible with the enhancement to $D_6 \times \mathrm{U}(1)^3$ in presence of an opportune set of flippers [40]. Analogously, the latter is compatible with an $E_7 \times \mathrm{U}(1)$ enhancement [33]. Here we have broken the nine dimensional Cartan of the $\mathrm{SU}(2n)$ gauge theory by introducing a 4d superpotential term between an antisymmetric and two fundamentals. Such a breaking has allowed us to state a 4d duality using the tensor deconfinement technique. Despite the fact that such a duality can be interesting *per se*, here we have studied its fate upon compactification on S^2 with non-negative integer R-charges through the prescription of [1]. We have found that $\mathrm{SU}(2n)$ with an antisymmetric flavor and four fundamental chirals is dual symplectic gauge theory is related to the LG theory expected from the analysis of [9]. In this sense the 4d duality obtained here is a parent of the 2d duality proposed in [9]. In the last part of the paper we obtained 2d dualities in presence of charged antisymmetric Fermi fields. Such dualities are especially interesting because it is not possible to deduce them from a pure 2d perspective by using the deconfining technique, or in other words, because we are not aware of any candidate *basic* 2d duality with Fermi fields in the antisymmetric representation of the flavor symmetry.

We conclude with some general comments on the IR dynamics. We expect that the gauge/LG dualities discussed in the paper give rise to interacting CFT in the IR. The reason behind such expectation is that in presence of non-compact target spaces the global symmetries associated to such non-compact directions get removed from the c-extremization procedure. In particular, the trial R-symmetry discussed in the various example is also the exact one. The central charge is then positive and there is no violation of the unitarity bounds. The presence of a non-compact target space implies also that we cannot turn off the fugacities in the matching of the elliptic genera. This signals that the dualities discussed here have to be interpreted as dualities in the mass deformed case, where we are only in presence of isolated vacua. Similar observations have been made in the literature for dualities with charged antisymmetric chiral fields in [7–9].

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A Conventions on the elliptic genus

In this appendix we fix the conventions used in the paper for the elliptic genus. We refer here to the elliptic genus in the NSNS sector, defined originally in [41, 42] (see [43, 44] for the definition in the RR sector [41, 42]). The index is given by

$$I(\vec{u};q) \equiv I(\vec{u}) \equiv \mathrm{Tr}_{\mathrm{NSNS}}(-1)^F q^{L_0} \prod_a u_a^{c_a}, \tag{A.1}$$

where $q = \mathrm{e}^{2\pi \mathrm{i}\tau}$ and τ refers to the complex structure of T^2. Formula (A.1) can be interpreted as a Witten index refined by the flavor fugacities u_a. Flat directions render the index divergent in presence of non-compact target space, therefore, it is crucial to keep the flavor fugacities turned on. This can be interpreted as a restriction of the analysis to the massive theory.

Denoting the gauge group as G, the elliptic genus can be computed as an integral over the Cartan torus of G, expressed in terms of the gauge fugacities z

$$I(u) = \frac{1}{|W|}\oint_{\mathrm{JK}} \prod_{i=1}^{\mathrm{rk}\,G} \frac{\mathrm{d}z_i}{2\pi \mathrm{i} z_i} I_V(\vec{z}) I_\chi(\vec{z},\vec{u}) I_\psi(\vec{z},\vec{u}), \tag{A.2}$$

where JK refers to the fact that the integral is computed by using the Jeffrey-Kirwan prescription.

The vector, the chiral and the Fermi multiplets contribute respectively as

$$\begin{aligned} I_V(\vec{z}) &= (q;q)_\infty^{2\mathrm{rk}\,G} \prod_{\alpha_G} \theta\left(z^{\alpha_G}\right), \\ I_\chi(\vec{z},\vec{u}) &= \prod_{\rho_G,\rho_F} \frac{1}{\theta\left(q^{\frac{R_\chi}{2}} z^{\rho_G} u^{\rho_F}\right)}, \\ I_\psi(\vec{z},\vec{u}) &= \prod_{\rho_G,\rho_F} \theta\left(q^{\frac{R_\psi+1}{2}} z^{\rho_G} u^{\rho_F}\right), \end{aligned} \tag{A.3}$$

with $\theta(x) = (x;q)_\infty (qx^{-1};q)_\infty$ and $(x;q)_\infty = \prod_{j=0}^\infty (1 - xq^j)$.

The index of an $\mathrm{SU}(N)$ gauge theory with F fundamentals Q, $\tilde{F}$ antifundamentals $\tilde{Q}$, H fundamental (or antifundamental) Fermi Λ, K antisymmetrics A and $\tilde{K}$ conjugate antisymmetrics $\tilde{A}$ can be expressed as

$$
\begin{aligned}
I_{\mathrm{SU}(N)}^{(F,\tilde{F};H;K;\tilde{K})}(\vec{m};\vec{n};\vec{h};\vec{r},\vec{s}) = {} & \frac{(q;q)_\infty^{2(N-1)}}{N!} \\
& \times \oint_{\mathrm{JK}} \prod_{i=1}^{N} \frac{\mathrm{d}z_i}{2\pi \mathrm{i} z_i} \frac{\prod_{i<j} \theta\left((z_i/z_j)^{\pm 1}\right) \prod_{i=1}^{N} \prod_{a=1}^{H} \theta(q^{\frac{R_\Lambda+1}{2}} z_i h_a)}{\prod_{i=1}^{N} \left(\prod_{a=1}^{F} \theta\left(q^{R_Q} z_i m_a\right) \cdot \prod_{a=1}^{\tilde{F}} \theta(q^{R_{\tilde{Q}}} z_i^{-1} n_a) \right)} \\
& \times \frac{\delta(1-\prod_{i=1}^{N} z_i)}{\prod_{i<j} \left(\prod_{a=1}^{K} \theta(q^{R_A} r_a z_i z_j) \cdot \prod_{a=1}^{\tilde{K}} \theta\left(q^{R_{\tilde{A}}} s_a z_i^{-1} z_j^{-1}\right) \right)} . \qquad \text{(A.4)}
\end{aligned}
$$

Throughout the paper, we have conventionally omitted the fugacities by leaving a $\cdot$ in the argument, when the associated field is absent.

The index of an $\mathrm{USp}(2N)$ gauge theory with F fundamentals Q, H fundamental Fermi multiplets Λ and one traceless antisymmetric chiral A is denoted as

$$
\begin{aligned}
I_{\mathrm{USp}(2N)}^{(F;H;1)}(\vec{m};\vec{h};r) = {} & \frac{(q;q)_\infty^{2N}}{2^N N!\,\theta\left(q^{r_A} r\right)^{N-1}} \oint_{\mathrm{JK}} \prod_{i=1}^{N} \frac{\mathrm{d}z_i}{2\pi \mathrm{i} z_i} \frac{\prod_{i<j} \theta\left(z_i^{\pm 1} z_j^{\pm 1}\right) \prod_{i=1}^{N} \theta(z_i^{\pm 2})}{\prod_{i<j} \theta\left(q^{r_A} z_i^{\pm 1} z_j^{\pm 1} r\right)} \\
& \times \prod_{i=1}^{N} \frac{\prod_{a=1}^{H} \theta\left(q^{\frac{R_\Lambda+1}{2}} z_i^{\pm 1} h_a\right)}{\prod_{a=1}^{F} \theta(q^{R_Q} z_i^{\pm 1} m_a)} . \qquad \text{(A.5)}
\end{aligned}
$$

References

[1] A. Gadde, S. S. Razamat and B. Willett, *On the reduction of 4d $\mathcal{N}=1$ theories on $\mathbb{S}^2$*, JHEP **11** (2015) 163 [1506.08795].

[2] F. Benini and A. Zaffaroni, *A topologically twisted index for three-dimensional supersymmetric theories*, JHEP **07** (2015) 127 [1504.03698].

[3] F. Benini and N. Bobev, *Exact two-dimensional superconformal R-symmetry and c-extremization*, Phys. Rev. Lett. **110** (2013) 061601 [1211.4030].

[4] F. Benini and N. Bobev, *Two-dimensional SCFTs from wrapped branes and c-extremization*, JHEP **06** (2013) 005 [1302.4451].

[5] N. Seiberg, *Electric - magnetic duality in supersymmetric nonAbelian gauge theories*, Nucl. Phys. B **435** (1995) 129 [hep-th/9411149].

[6] K. A. Intriligator and P. Pouliot, *Exact superpotentials, quantum vacua and duality in supersymmetric SP(N(c)) gauge theories*, Phys. Lett. B **353** (1995) 471 [hep-th/9505006].

[7] M. Sacchi, *New 2d $\mathcal{N}$ = (0, 2) dualities from four dimensions*, JHEP **12** (2020) 009 [2004.13672].

[8] J. Jiang, S. Nawata and J. Zheng, *2d dualities from 4d*, 2407.17350.

[9] A. Amariti, P. Glorioso, F. Mantegazza, D. Morgante and A. Zanetti, *Dualities from dualities in 2d $\mathcal{N}=(0,2)$*, 2410.12453.

[10] M. Berkooz, *The Dual of supersymmetric SU(2k) with an antisymmetric tensor and composite dualities*, Nucl. Phys. B **452** (1995) 513 [hep-th/9505067].

[11] M. A. Luty, M. Schmaltz and J. Terning, *A Sequence of duals for Sp(2N) supersymmetric gauge theories with adjoint matter*, Phys. Rev. D **54** (1996) 7815 [hep-th/9603034].

[12] S. Pasquetti and M. Sacchi, *From 3d dualities to 2d free field correlators and back*, JHEP **11** (2019) 081 [1903.10817].

[13] S. Benvenuti, I. Garozzo and G. Lo Monaco, *Monopoles and dualities in 3d$\mathcal{N}$ = 2 quivers*, JHEP **10** (2021) 191 [2012.08556].

[14] I. G. Etxebarria, B. Heidenreich, M. Lotito and A. K. Sorout, *Deconfining $\mathcal{N}$ = 2 SCFTs or the art of brane bending*, JHEP **03** (2022) 140 [2111.08022].

[15] S. Benvenuti and G. Lo Monaco, *A toolkit for ortho-symplectic dualities*, JHEP **09** (2023) 002 [2112.12154].

[16] L. E. Bottini, C. Hwang, S. Pasquetti and M. Sacchi, *Dualities from dualities: the sequential deconfinement technique*, JHEP **05** (2022) 069 [2201.11090].

[17] S. Bajeot and S. Benvenuti, *Sequential deconfinement and self-dualities in 4d$\mathcal{N}$ = 1 gauge theories*, *JHEP* **10** (2022) 007 [2206.11364].

[18] S. Bajeot and S. Benvenuti, *4d$\mathcal{N}$ = 1 dualities from 5d dualities*, *JHEP* **08** (2024) 197 [2212.11217].

[19] A. Amariti and S. Rota, *3d N=2 SO/USp adjoint SQCD: s-confinement and exact identities*, *Nucl. Phys. B* **987** (2023) 116068 [2202.06885].

[20] A. Amariti, F. Mantegazza and D. Morgante, *Sporadic dualities from tensor deconfinement*, *JHEP* **05** (2024) 188 [2307.14146].

[21] A. Amariti and F. Mantegazza, *A new 4d $\mathcal{N}$ = 1 duality from the superconformal index*, *JHEP* **06** (2024) 206 [2402.00609].

[22] A. Amariti and F. Mantegazza, *Confinement for 3d $\mathcal{N} = 2SU(N)$ with a Symmetric tensor*, 2405.11972.

[23] S. Benvenuti, R. Comi, S. Pasquetti and M. Sacchi, *Deconfinements, Kutasov-Schwimmer dualities and $D_p[SU(N)]$ theories*, 2407.11134.

[24] C. Hwang and S. Kim, *S-confinement of 3d Argyres-Douglas theories and the Seiberg-like duality with an adjoint matter*, 2407.11129.

[25] S. Bajeot and S. Benvenuti, *S-confinements from deconfinements*, *JHEP* **11** (2022) 071 [2201.11049].

[26] C. Csaki, M. Schmaltz and W. Skiba, *Confinement in N=1 SUSY gauge theories and model building tools*, *Phys. Rev. D* **55** (1997) 7840 [hep-th/9612207].

[27] K. Nii, *On s-confinement in 3d $\mathcal{N} = 2$ gauge theories with anti-symmetric tensors*, 1906.03908.

[28] A. Amariti, F. Mantegazza and S. Rota, *Rank-two tensors and deconfinement in $su(n)$ gauge theories with 4 supercharges*, *To Appear* (2025) .

[29] T. Dimofte, D. Gaiotto and N. M. Paquette, *Dual boundary conditions in 3d SCFT's*, *JHEP* **05** (2018) 060 [1712.07654].

[30] V. P. Spiridonov and G. S. Vartanov, *Elliptic Hypergeometry of Supersymmetric Dualities*, *Commun. Math. Phys.* **304** (2011) 797 [0910.5944].

[31] O. Aharony, S. S. Razamat, N. Seiberg and B. Willett, *3d dualities from 4d dualities*, *JHEP* **07** (2013) 149 [1305.3924].

[32] M. Dedushenko and S. Gukov, *IR duality in 2D $N = (0,2)$ gauge theory with noncompact dynamics*, *Phys. Rev. D* **99** (2019) 066005 [1712.07659].

[33] S. S. Razamat and G. Zafrir, *E_8 orbits of IR dualities*, *JHEP* **11** (2017) 115 [1709.06106].

[34] V. P. Spiridonov and S. O. Warnaar, *Inversions of integral operators and elliptic beta integrals on root systems*, Advances in Mathematics **207** (2006) 91.

[35] B. Nazzal, A. Nedelin and S. S. Razamat, *Minimal (D, D) conformal matter and generalizations of the van Diejen model*, SciPost Phys. **12** (2022) 140 [2106.08335].

[36] F. J. van de Bult, *An elliptic hypergeometric beta integral transformation*, *arXiv preprint arXiv:0912.3812* (2009) .

[37] C. Closset and I. Shamir, *The $\mathcal{N} = 1$ Chiral Multiplet on $T^2 \times S^2$ and Supersymmetric Localization*, JHEP **03** (2014) 040 [1311.2430].

[38] M. Honda and Y. Yoshida, *Supersymmetric index on $T^2 \times S^2$ and elliptic genus*, 1504.04355.

[39] T. Okazaki and D. J. Smith, *Boundary confining dualities and Askey-Wilson type q-beta integrals*, JHEP **08** (2023) 048 [2305.00247].

[40] S. S. Razamat, O. Sela and G. Zafrir, *Curious patterns of IR symmetry enhancement*, JHEP **10** (2018) 163 [1809.00541].

[41] A. Gadde and S. Gukov, *2d Index and Surface operators*, JHEP **03** (2014) 080 [1305.0266].

[42] A. Gadde, S. Gukov and P. Putrov, *Walls, Lines, and Spectral Dualities in 3d Gauge Theories*, JHEP **05** (2014) 047 [1302.0015].

[43] F. Benini, R. Eager, K. Hori and Y. Tachikawa, *Elliptic genera of two-dimensional N=2 gauge theories with rank-one gauge groups*, Lett. Math. Phys. **104** (2014) 465 [1305.0533].

[44] F. Benini, R. Eager, K. Hori and Y. Tachikawa, *Elliptic Genera of 2d $\mathcal{N} = 2$ Gauge Theories*, Commun. Math. Phys. **333** (2015) 1241 [1308.4896].