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On a modified quantum theory with objective quantum thermalization and spontaneous universal irreversibility

Aritro Mukherjee¹

¹Institute for Theoretical Physics Amsterdam, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands

The deterministic and time-reversal symmetric dynamics of isolated quantum systems is at odds with irreversible equilibration observed in generic thermodynamic systems. Standard approaches at a reconciliation are based on agent-specific restrictions on the space of observables or states and do not explain how a single macroscopic quantum system achieves equilibrium dynamically. We instead argue that quantum theory is an effective theory and requires corrections to accurately describe systems approaching the thermodynamic limit. We construct a minimal extension of quantum theory which is practically identical to quantum mechanics for microscopic systems, yet allows isolated, macroscopic systems to thermalize, with an objective notion of thermalization. A fluctuation-dissipation relation guarantees physicality constraints including norm preservation, energy conservation, no superluminal signalling and the emergence of microcanonical equilibrium statistics. We further discuss the inclusion of objective collapse, thereby realizing a falsifiable theory of spontaneous universal irreversibility which describes the quantum-to-classical crossover dynamics of macroscopic quantum systems. This model admits spontaneous symmetry breaking, quantum state reduction and objective quantum thermalization for individual systems while realizing an emergent hybrid, Born-Maxwell-Boltzmann-Gibbs-microcanonical distribution for ensembles.

Introduction — The irreversible approach of physical systems towards equilibrium is ubiquitous in nature and underlies the widespread success of equilibrium statistical mechanics at all length scales [1-3]. However, how equilibrium is achieved dynamically, within closed quantum systems undergoing time-reversal symmetric and deterministic dynamics (such as via Schrödinger's equations) remains a foundational open question [4-6], which we term the quantum thermalization problem (QTP).

The unitary evolution of the Schrödinger equation ensures that probability amplitudes do not change in the energy basis, up to a dynamical phase. This implies that the memory of the initial state is preserved and that the wavefunction remains time dependent and cannot converge to any time-independent equilibrium state. Thus, although desirable, the strongest notion of an irreversible approach to equilibration for an isolated system, one at the level of the (pure) quantum state, valid for all possible observables of the given system, is trivially disallowed within standard quantum dynamics [4–6].

Within the confines of standard quantum theory only various effective or weaker notions of irreversibility are tenable, which generally propose restrictions on the space of states or observables. These restrictions are motivated via epistemological arguments based on agent-specific considerations as to which observables are 'viable' or 'physical' and proceed via prescribing a subset $\mathcal{A}_R \subset \mathcal{A}$ as relevant, out of all possible quantum observables in \mathcal{A} . Confined to this restricted algebra of observables, \mathcal{A}_R , such approaches are confined to ensemble averaged insights using coarse-grained mixed density operators (normal states on the subset \mathcal{A}_R are generically mixed [7]) and such density matrices do not describe how a single system undergoes irreversible dynamics [8, 9]. Further, since these restrictions are based on epistemic motivations, such as what may or may not be accessible to a given agent, different restricted sets \mathcal{A}_R and \mathcal{A}'_R may be seen as viable choices of observables for different agents. Although specific agents may interact with nature in specifically constrained ways, one cannot assume that nature interacts with *itself* in any particularly constrained way. Thus trivially, physical systems behave independent of what is practically knowable in a particular situation. Therefore, such restricted sets of observable algebras cannot be utilized to construct any objective notion of irreversibility or equilibration, a notion which should remain valid for all possible observables and hence all possible agents investigating a given quantum system.

In this article, we take the view that quantum theory, specifically its dynamical equations develop significant corrections or modifications for systems with large number of degrees of freedom, i.e. for systems approaching the thermodynamic limit. Our motivation derives from objective collapse theories [9-22], which aim at resolving the quantum measurement problem via modifications of quantum theory. The irreversible and random phenomenon of quantum state reduction or wavefunction collapse during measurements (with physical devices) also cannot be accounted for within the timereversal symmetric equations, such as the non-relativistic Schrödinger equation, and constitutes the quantum measurement problem [7, 9, 10, 23, 24]. Objective collapse theories resolve this problem by introducing small modifications to Schrödinger's equation in such a way that the unitary time evolution of microscopic particles is unaffected in any noticeable way, while the effect of the modifications dominate the dynamics in the macroscopic regime and cause quantum superpositions of large objects to reduce to classical configurations. In this context, quantum systems approaching the thermodynamic limit are also understood to be in the quantum-to-classical crossover regime: a regime where classicality is expected to emerge from the underlying quantum theory of its constituents.

A typical objective collapse theory, however, cannot lead to any universal notion of thermalization, since the information of the initial conditions of the entire closed system remain partially preserved via the so called martingale condition, allowing the emergence of Born's statistics [10, 17, 19]. Following this line of thought, corrections leading to objective collapse and those corrections leading to thermalization may be treated separately [9], the latter being the focus of this article.

After a brief discussion of thermalization in classical and quantum systems, we review the strongest possible notion of quantum thermalization, that at the level of pure states and we discuss the requirements of a modified quantum theory of objective quantum thermalization (OQT). A generic form of an OQT model is argued, and we demonstrate how a fluctuation-dissipation relation enforces physicality conditions, such as norm preservation, and ensures the absence of superluminal signalling. Ensembles of systems are shown to evolve via a (linear) quantum semi-group.

The ensemble expectations of the energy and the long time steady states of the OQT model are then used to extract stringent constraints to the theory. Using these constraints, a unique form of the OQT model is established which allows systems to equilibrate to a microcanonical distribution, with a notion of equilibration that is valid for all possible observables. The dynamical emergence of microcanonical equilibrium distributions, entropy increase and the conservation of average energy are ensured within the constrained model. We further discuss a concrete thought experiment constructing protocols, attempting to signal faster than light using the OQT model and show that it is disallowed in the model, given a certain locality condition.

Finally, we consider the integration of the OQT model with the previously established objective collapse models. The OQT model and objective collapse models are shown to play complimentary roles, resolving both the quantum measurement problem and the quantum thermalization problem within the same theory. We term these models of Spontaneous Universal Irreversibility (SUI) because they constitute a minimal modification of quantum dynamics which allow macroscopic systems to spontaneously exhibit irreversible and stochastic behaviour and approach classical equilibrium states.

Focusing on a recently proposed objective collapse model which extends spontaneous symmetry breaking to the dynamical regime, we show that the hybrid SUI model describes three well known spontaneous irreversible phenomena for thermodynamic quantum systems—quantum state reduction, spontaneous symmetry breaking and objective thermalization. Our results open up new possibilities of observational tests of fundamental physics and we hope to motivate a critical re-analysis of quantum interpretations and the foundations of equilibrium statistical mechanics.

Classical Considerations—Already in the classical regime, epistemic restrictions are employed to explain how physical systems approach equilibrium. An isolated classical system evolves under deterministic, time-reversal symmetric Hamiltonian dynamics, which by itself cannot lead to any irreversibility; moreover, spatially bound systems exhibits Poincaré recurrences, returning arbitrarily close to its initial state infinitely many times [2, 3, 25, 26]. Both these issues, the problem of reversibility and the problem of recurrences [2, 25], reappear in the quantum regime for isolated systems [4–6].

Classical equilibration stands on two pillars—Gibb's ensemble approach and Boltzmann's single system approach [2]. Both approaches rely heavily on long time averages and coarse-graining the phase (state) space to allow (mixed) densities instead of (pure) delta measures and also, coarse-graining the space of observables, motivated by what may be reliably known in a specific scenario, to a specific agent [2–5, 27, 28].

In the Gibbs ensemble approach, coarse (mixed) probability densities (instead of delta measures) are employed, implying a lack of knowledge of the system's finer details or an uncertainty in the initial conditions. This is augmented with further coarse-graining of the phase space cells itself, restricting the investigation to macroscopic observables—smooth functions on the coarse phase space. Finally, one must also argue some manner of ergodicity [2, 3, 27]—that long time averaged quantities approach phase space ensemble averages under an appropriately coarse-grained thermal probability density or time invariant measure. However, physical systems where equilibration is expected, have not been definitively proved to possess constrained conditions such as ergodicity or more stronger properties like mixing [2, 27]. Usually, ergodicity with an appropriate coarse-grained measure, is justified by focusing on particular systems, most notably classical chaotic systems which showcase exponential divergent trajectories of nearly identical initial conditions [2, 3]. This allows a notion of ensembles of systems approaching equilibrium, however, this notion is not valid for all possible observables of a given system. Thus, different agents with differing coarse-graining of the phase space and possessing different observables may not agree on what constitutes equilibrium.

Boltzmann provided the first microscopic derivation of the second law of thermodynamics, by establishing the celebrated H-theorem—showing the increase in entropy in a classical ideal gas—while making the so called *Stosszahlansatz* or molecular chaos assumption, neglecting correlations [2, 3, 25, 28]. However, following arguments by Loschmidt and Zermelo that the assumption is not consistent with the time-reversal symmetric and deterministic Hamiltonian laws of classical mechanics, Boltzmann developed two connected ideas arguing for the increase of entropy—the notion of typicality and the idea that the universe started from a lower entropic state [2, 3].

Boltzmann's typicality observes that the phase space may be decomposed into sectors, where the observable expectation values of each microstate in the largest sectors converge to a thermal expectation, for certain 'viable' macroscopic observables. Thus, typically, the systems spend the largest time traversing the overwhelmingly large *equilibrium* sectors where these macroscopic observables have appropriate thermal expectation values. Note that such statements do not forbid a system from spending timescales as large as the age of the universe out of equilibrium, before entering an equilibrium sector [29, 30]. Further, the fact that such equilibrium sectors dominate the phase space, does not imply that nonequilibrium states do not exist; in fact they may be prepared in laboratories and in some sense, the dynamical world at large, including biological systems, are indeed out of equilibrium [2, 25, 26]. Mathematical statements on the non-equilibrium sectors approaching measure zero in the thermodynamic limit, do not imply their negligible contribution to the physical world at large [2, 25]. Finally, to account for the observed increase in entropy, one further argues that the universe started in a low entropy state, such that there many more ways of reaching a higher entropic state [2, 3, 25].

Note crucially that the expectation values of only a restricted set of observables, converge to thermal expectation values, and such considerations allow at best an agent-dependent notion of what seems to be at equilibrium. One agent restricted to certain energy and time scales, determining their coarse-graining and chosen macroscopic observables, may not agree with other agents with access to more fine-grained observables. Thus, in both the above approaches, the increase in entropy, the approach to equilibrium and ultimately the derived thermodynamics are a consequence of the lack of knowledge of physical agents, not a fundamental property of the physical systems in and of itself.

Quantum Conundrums—Similar issues persist in the quantum case, and the arguments used towards a resolution of the QTP. Early works by von Neumann extending the notion of ergodicity and typicality to the quantum case, already suggest the requirement of restricting the space of observables (allowing only macroscopic observables which commute) and the state space, in addition to other requirements such as no resonances [30–32]. Consequent works by the Austin-Brussels group, although motivated by similar goals as this article—to universally account for irreversibility—focused on special

systems and observables while employing a strict ensemble worldview, abandoning the treatment of single systems [26, 33–36].

The Zubarev school also attempted to explain quantum equilibration by instead focusing on non-standard modifications of the master equations [37], however it is now well known that non-standard, especially non-linear modifications of the master equations may result in superluminal signalling [9, 38–40]. Other approaches employ the quantum Boltzmann equations which focus on neglecting correlations (like the classical Boltzmann approach) and approximating the analogous quantum collision terms in the master equations [41, 42]. Note that both approaches employ epistemic restrictions on the space of observables and neglect single systems.

Contemporary approaches towards a resolution of the QTP, such as the eigenstate thermalization hypothesis (ETH)[4–6, 43–46], coupled with insights from open quantum systems and the decoherence paradigm [47–49], both, prescribe a restricted set of observables as 'physically accessible' and admit purely agent-dependent notions of irreversibility. Again, such notions are only valid for specific, coarse and non-uniquely decomposable ensembles (with epistemic restrictions) and cannot constitute an agent independent or objective notion of equilibrium (for all possible observables), nor can it explain how a single quantum system can undergo irreversible behaviour.

Decoherence [47, 48] based approaches towards a resolution, focus on ensembles of a subsystem, or equivalently a restriction of observables to only subsystem observables $\mathcal{A}_S \subset \mathcal{A}$. Normalized states on these restricted set of observables (\mathcal{A}_S) are generically mixed sub-system density operators, corresponding to the marginalized state of the subsystem averaged over all possible environment states. These mixed, coarse-grained densities are non-uniquely decomposable and are confined to ensembles, which trivially implies their inability to describe a *single* instance of irreversible evolution. In other words, decoherence does not explain how a single setup can undergo irreversible and random dynamics [8, 9, 17, 50]. Further a different choice of sub-systems may showcase the presence of correlations and persistent memory of the initial conditions, thus leading to various agents (with access to different subsystems) disagreeing on what constitutes equilibrium.

Recently, the eigenstate thermalization hypothesis (ETH) has been subject to extensive investigations. ETH considers only those observables viable or physical which, in the energy basis, possess smoothly varying diagonal elements, while off-diagonals are suppressed and scale inversely with the system size [4–6, 43, 44, 46]. These preferred, physically viable, coarse-grained observables, $\mathcal{A}_E \subset \mathcal{A}$ may *seem* thermalized, while other observables in \mathcal{A} will trivially never thermalize. The expectation values of these ETH-observables (\mathcal{A}_E), for most states in the Hilbert space confined to a narrow interval of energy,

approximately converge on thermalized expectations and more so if the observable diagonal elements do not appreciably change within the energy window of interest.

Notions of quantum typicality—that almost every pure state yields thermal expectations for coarse macroscopic observables-are further used to augment ETH, yet again do not realize an agent-independent notion of thermalization, one valid for all possible observables of a system [6, 29, 51, 52]. Further, restricting to certain coarse algebra of observables, imply that normal states on these algebras are mixed [7], i.e. the density matrices are mixed and non-uniquely decomposable, which describe only ensembles and thus how an individual quantum system in a pure micro-state undergoes an irreversible approach to equilibrium is not described. Note, observables such as $|E\rangle\langle E'|+|E'\rangle\langle E|\in\mathcal{A}$ with large off-diagonals in the energy basis $(|E\rangle, |E'\rangle)$ are not ETH observables, and they remain coherent and do not equilibrate within standard quantum theory. It is always possible to construct such ETH-violating observables which are typically argued away as being un-physical or inaccessible (to specific agents). Thus again ETH does not constitute an agent-independent notion of equilibration. Further, it is well known than ETH is evaded in many systems such as in integrable systems [4, 5], systems with quantum scarring [53] and many-body localization [46, 54].

Recently, it has been shown that given the initial conditions and observables, the question of whether any given Hamiltonian yields thermalized expectations or not, can be mapped onto the Turing halting problem, and is thus undecidable within the mathematical framework of quantum theory and its axioms [55]. These issues further motivate us to explore a different route via modifying the standard axioms and instead focus on falsifiable effects of such modifications.

In summary, the various notions of quantum thermalization admitted within standard quantum theory are agent dependent notions and do not explain how a single quantum system can undergo irreversible dynamics. Thus our main goals are to construct a viable notion of single system irreversibility and an agent-independent notion of equilibration. We will show that acknowledging the possibility that quantum theory is effective and may require corrections beyond the interval of its validity, i.e. for systems approaching the thermodynamic limit, allows us to recover a stringently constrained modified quantum theory, which achieves both these goals, while being experimentally falsifiable.

Objective quantum thermalization — In view of modifying quantum theory to allow irreversible dynamics, objective collapse theories motivate our main approach. Objective collapse theories modify the quantum dynamical evolution such that they are practically indistinguishable from quantum theory for microscopic systems, while for macroscopic systems allow an objective notion of quantum state reduction or wave-function collapse, applicable for all observables of an isolated system [10, 17–21, 56]. Crucially, ensembles of isolated systems undergoing objective collapse dynamics, at long times, follow a dynamical map, $\hat{\rho} \rightarrow \hat{\rho}_{\rm B}$ where $\hat{\rho}_{\rm B}$ is a statistical distribution corresponding to Born's rules. In an analogous way, we are specifically interested in a modified quantum theory which can admit a dynamical map for ensembles, corresponding to the strongest notion of thermalization for isolated systems, wherein the state itself thermalizes, $\hat{\rho} \rightarrow \hat{\chi}$, where $\hat{\chi}$ is a thermalized density matrix corresponding to an equilibrium distribution attained by an ensemble of isolated systems, such as the microcanonical distribution.

This strongest notion of thermalization is trivially not admissible via the unitary dynamics of Schrödinger's equation which preserves the memory of the initial state. Indeed, our approach, in contrast to previous approaches focus on explicitly changing the dynamics of isolated quantum systems, which allow an objective resolution of the QTP, independent of any choice of (thermodynamic) systems, initial states or preferred observables. Further, we ensure that the modifications scale with the system size and thus for small systems, the modified dynamics is practically indistinguishable from quantum theory while large systems thermalize objectively. Due to the differing dynamics of large quantum systems from its standard expectations, such theories are in principle falsifiable via direct experimentation in the mesoscopic regime, similar to objective collapse theories [22, 56-58]

We will now construct a model allowing objective thermalization for a macroscopic, isolated, quantum system with Hamiltonian H. We consider a (not necessarily finite) countable Hilbert space, \mathcal{H} with energy eigenstates, $\hat{H} |\mu\rangle = E_{\mu} |\mu\rangle$. To construct the model we first argue that unlike objective collapse models which possess the so called martingale property [10, 17, 19], any objective thermalization model cannot posses this martingale property in the energy basis, since this would preserve the information of the initial conditions [9]. Thus, a viable starting point for the so-called thermalization operators in any modified quantum theory allowing an objective notion of thermalization, must present possible transitions between energy eigenstates. The primitive operator allowing such transitions is $\hat{L}_{\mu\nu} = |\mu\rangle \langle \nu|$, which determines the form of an un-normalized quantum (Ito) stochastic process [10, 38, 59] on \mathcal{H} , given by ($\hbar = 1$) :

$$d |\psi\rangle = -i\hat{H} |\psi\rangle dt + \sum_{\mu,\nu} d\hat{G}^{\mu\nu} |\psi\rangle , \qquad (1)$$

$$d\hat{G}^{\mu\nu} |\psi\rangle := D^{\mu\nu} \left[\hat{L}_{\mu\nu} - \langle \hat{L}_{\mu\nu} \rangle \right] |\psi\rangle dW_t^{\mu\nu} + J^{\mu\nu} \left[\langle \hat{L}^{\dagger}_{\mu\nu} \rangle \hat{L}_{\mu\nu} - \frac{1}{2} \hat{L}^{\dagger}_{\mu\nu} \hat{L}_{\mu\nu} - \frac{1}{2} \langle \hat{L}_{\mu\nu} \rangle \langle \hat{L}^{\dagger}_{\mu\nu} \rangle \right] |\psi\rangle dt.$$

Here, $|\psi\rangle$ is the time dependent wave function and \hat{H} is the standard Hamiltonian, while $d\hat{G}^{\mu\nu}$ is the stochas-

tic modification, to be constrained further. If $d\hat{G}^{\mu\nu}$ = $0, \forall \mu, \nu, \text{ Eq. } (1)$ reduces to the standard Schrödinger equation evolving isolated systems. Here $\langle \hat{O} \rangle = \langle \psi | \hat{O} | \psi \rangle$ is the usual (time dependent) quantum expectation value for an individual system in the state $|\psi\rangle$.

To ensure that these modifications are significant for macroscopic quantum objects, the strength of the modifications must scale with the (effective) number of degrees of freedom of the system implying that $d\hat{G}^{\mu\nu}$ is extensive and proportional to a phenomenological coupling \mathcal{N} . Within each $d\hat{G}^{\mu\nu}$ there are two terms controlling the rate of transitions, one is non-linear and deterministic, with the coupling $J^{\mu\nu}$, while the other term is stochastic, coupled via $D^{\mu\nu}$. Both terms must be proportional to \mathcal{N} , as further clarified by the fluctuation dissipation relationship discussed below. Note that both coefficients are non-negative real-valued numbers, and are not symmetric, i.e. $J^{\mu\nu} \neq J^{\nu\mu}$ and $D^{\mu\nu} \neq D^{\nu\mu}$.

The factor $dW_t^{\mu\nu}$ indicates real valued Gaussian increments of independent standard Wiener processes (i.e. corresponding to the Brownian motion $W_t = \int dW_t$ with $W_0 = 0$ [60, 61]. In the above expression note that $dW_t^{\mu\nu}$ and $dW_t^{\nu\mu}$ are increments of independent Wiener processes. We use the convention that each $dW_t^{\mu\nu}$ is sampled from a Gaussian distribution with standard deviation \sqrt{dt} and implies the standard time independent expectation values, $\mathbb{E}\left[dW_t^{\mu\nu}\right] = \mathbb{E}\left[W_t^{\mu\nu}\right] = \mathbb{E}\left[dt \, dW_t^{\mu\nu}\right] = 0$, $\mathbb{E}\left[dW_t^{\mu\nu} \, dW_s^{\mu\nu}\right] = 0$ for $t \neq s$ and at equal times, $\mathbb{E}\left[dW_t^{\mu\nu} \, dW_t^{\mu'\nu'}\right] = \delta_{\mu\mu'} \delta_{\nu\nu'} dt$ or simply, $(dW_t^{\mu\nu})^2 = dt$ [60, 61]. Here $\mathbb{E}[\cdot]$ indicates the average over an ensemble of realizations of the Wiener process and hence, trajectories of Eq. (1). Note that we assume that this stochastic influence arises from physics beyond quantum mechanics, and it is not an averaged description of the influence of unobserved quantum degrees of freedom.

We now consider constraints such that the modifications do not change the kinematic character of quantum theory. Particularly, norm-conservation is upheld in the dynamics of Eq. (1) by observing a fluctuation dissipation relationship (FDR) between the stochastic and deterministic components of the modifications [17, 19]. Concretely, using the norm, $N_{\psi} = \langle \psi | \psi \rangle$ and applying Ito's lemma, $dN_{\psi} = \langle d\psi | \psi \rangle + \langle \psi | d\psi \rangle + \langle d\psi | d\psi \rangle$, we find its change, $dN_{\psi} = 0$ (at all times) with an FDR of the form $(D^{\mu\nu})^2 = J^{\mu\nu} \propto \mathcal{N}$. This implies that within an ensemble of identically prepared systems, each ensemble member individually undergoes the stochastic and normconserved dynamics of Eq. (1) while the FDR is upheld.

To find how the ensemble evolves we consider the evolution of the pure state projector, $\hat{\Psi} := |\psi\rangle \langle \psi|$, which describes how a single ensemble member evolves. Here the ensemble constitutes individual systems evolving via Eq. (1) with differing stochastic trajectories. The stochastic average over these, $\mathbb{E}[\hat{\Psi}] = \hat{\rho}$, yields how the entire ensemble density evolves. To compute this, we

use Ito's lemma, $d\hat{\Psi} = |d\psi\rangle\langle\psi| + |\psi\rangle\langle d\psi| + |d\psi\rangle\langle d\psi|$, and average over the stochastic components to obtain the evolution of the corresponding statistical operator or density matrix, with $\mathbb{E}[d\Psi] = d\hat{\rho}$. This yields the linear master equations of the Gorini-Kossakowsky-Sudarshan-Lindblad (GKSL) form [49, 62, 63], given by $(\hbar = 1)$:

$$\frac{\partial \hat{\rho}}{\partial t} = -i \left[\hat{H}, \, \hat{\rho} \right]
+ \mathcal{N} \sum_{\mu,\nu} J^{\mu\nu} \left(\hat{L}_{\mu\nu} \hat{\rho} \hat{L}^{\dagger}_{\mu\nu} - \frac{1}{2} \left\{ \hat{L}^{\dagger}_{\mu\nu} \hat{L}_{\mu\nu}, \, \hat{\rho} \right\} \right). \quad (2)$$

Here $\hat{\rho}$ is the noise averaged statistical operator (density matrix) for an ensemble of systems undergoing the dynamics of Eq. (2) with the FDR, $(D^{\mu\nu})^2 = J^{\mu\nu}$. The proportionality factor scaling with the system size, \mathcal{N} has been extracted from the definition of the coupling constants. Note that the above master equation, unlike those obtained in objective collapse theories, resembling dephasing GKSL equations [10, 17, 19], takes a form explored in the context of open system equilibration via detailed balance [64, 65]. This will allow us to constrain the model further so that an appropriate steady state is reached, given the system Hamiltonian. However, we stress here that in this model, a (linear) quantum semigroup is obtained for the dynamics of an ensemble of isolated systems and unlike other approaches, does not integrate out inaccessible parts of the setup. Instead the master equations results from averaging over the modified stochastic quantum dynamics for each ensemble member, which are single instances of isolated systems undergoing the dynamics of Eq. (1) with the FDR.

Equilibrium, Energy and Entropy Constraints— Having obtained the ensemble dynamics of isolated quantum systems in Eq. (2), we will now show that constraints arising from its equilibrium steady states and the average energy conditions, impose stringent constraints on the model and the form of $J^{\mu\nu}$. This will allow us to construct an energy conserving theory of objective quantum thermalization viable for isolated systems.

Our main goal is ensure that for any (non-equilibrium) state $\hat{\rho}$, the dynamics of Eq. (2) guarantees that a steady equilibrium state is reached at long times. Denote these steady states corresponding to an equilibrium density (statistical) operator as $\hat{\chi}$, which is time independent, mixed and diagonal in the energy basis. $\hat{\chi}$ is diagonal since it represents an average over an ensemble and offdiagonal contributions are necessarily averaged out, as noted in both the quantum microcanonical ensemble and canonical ensemble, which are diagonal in the energy basis [1, 4-6, 29, 52]. We will obtain these steady states as solutions of the constraints below, however it is important to note that $\hat{\chi}$ ultimately represents the physically observed equilibrium distribution, reached by an ensemble of identical, isolated (macroscopic) quantum systems. Imposing the steady state condition, $\frac{\partial \hat{\rho}}{\partial t} = 0$, in Eq. (2),

we find the first (operator) constraint, $\hat{\mathcal{C}}_{\chi} = 0$, towards achieving the equilibrium steady state $\hat{\chi}$, given by:

$$\hat{\mathcal{C}}_{\chi} := \sum_{\mu,\nu} J^{\mu\nu} \chi_{\nu} \bigg(|\mu\rangle \langle \mu| - |\nu\rangle \langle \nu| \bigg).$$

Here, $\chi_{\nu} = \langle \nu | \hat{\chi} | \nu \rangle$ and $\sum_{\nu} \chi_{\nu} = 1$ since $\text{Tr}[\hat{\chi}] = 1$. Phrased differently, if the dynamics of Eq. (2) leads to a long time equilibrium steady state of the form $\hat{\chi}$, then the constraint $\hat{\mathcal{C}}_{\chi} = 0$ must be satisfied.

Consider now the change in the average energy of the ensemble using Eq. (2) and $E = \text{Tr}[\hat{\rho} \hat{H}]$. Let the (standard) Hamiltonian be $\hat{H} = \sum_{\mu} E_{\mu} |\mu\rangle \langle \mu|$. The constraint ensuring energy conservation while describing thermalization in ensembles of isolated systems is $C_E(t) = 0$ where $C_E(t) := dE/dt$. In other words, for an ensemble of isolated systems undergoing the dynamics of Eq. (2), the average energy is conserved if and only if the constraint, $C_E(t) = 0$ is satisfied at all times, and for all states on the Hilbert space. Without assuming any restriction on $\hat{\rho}$, this constraint takes the form:

$$\mathcal{C}_E(t) := \sum_{\mu,\nu} J^{\mu\nu} \rho_{\nu\nu} \left(E_\mu - E_\nu \right)$$

Here, $\rho_{\mu\nu} = \langle \mu | \hat{\rho} | \nu \rangle$ is time dependent and in general, may be any state. Since any physically viable, objective thermalization model cannot allow for unrestricted changes in ensemble averaged energies, but must yield a long time equilibrium state, we must solve for both $\hat{\mathcal{C}}_{\chi} = 0$ and $\mathcal{C}_E(t) = 0$ together and obtain a form of $J^{\mu\nu}$. To do this, first we using an ansatz, $J^{\mu\nu} = A^{\mu}B^{\nu}$ in $\hat{\mathcal{C}}_{\chi} = 0$ and $\mathcal{C}_E(t) = 0$, which yield the following two suggestive expressions respectively:

$$\hat{C}_{\chi}: \sum_{\mu} A^{\mu} \sum_{\nu} \chi_{\nu} B^{\nu} \left(|\mu\rangle \langle \mu| - |\nu\rangle \langle \nu| \right) = 0,$$

$$\mathcal{C}_{E}(t): \sum_{\mu} A^{\mu} \sum_{\nu} B^{\nu} (E_{\mu} - E_{\nu}) \rho_{\nu\nu} = 0.$$

It is seen that both conditions may be uniquely met by the asymmetric choice of $B^{\nu} = 1$ yielding $A^{\mu}/\mathcal{Z} = \chi_{\mu}$ with $\sum_{\mu} A^{\mu} = \mathcal{Z}$, observed from the promising equalities:

$$\hat{C}_{\chi}: \quad \sum_{\mu} \left(\frac{A^{\mu}}{\mathcal{Z}} - \chi_{\mu} \right) \ |\mu\rangle \langle \mu| = 0, \qquad (3)$$

$$\mathcal{C}_E(t): \quad \sum_{\mu} E_{\mu} \frac{A^{\mu}}{\mathcal{Z}} = \sum_{\nu} E_{\nu} \rho_{\nu\nu}. \tag{4}$$

The first equality encodes the requirement that the Hilbert space of the quantum system must possess a time invariant thermal state $\hat{\chi}$ and is related to the quantum semi-group in Eq. (2) via $J^{\mu\nu} = A^{\mu} = \chi_{\mu}$ ($\forall \mu, \nu$ with $\mathcal{Z} = 1$). The second equality ensures energy conservation in the ensembles, and shows again that the A^{μ}

must be weights of a probability distribution. Note that while the right hand side is time dependent and equals $\text{Tr}[\hat{\rho}\hat{H}]$, the left hand side is independent of time, with $\sum_{\mu} E_{\mu} A^{\mu} = \text{Tr}[\hat{\chi}\hat{H}]$, using Eq. (3). Then, if the condition in Eq. (4) holds at initial times, it will hold for all times since Eq (2) is a quantum semi-group.

However, multiple solutions of $\hat{\chi}$ are possible which satisfy the above constraints. This is not unexpected given the fact that in a differing setting of (standard) open quantum systems, Eq. (2) may be interpreted as the reduced dynamics of a sub-system after averaging out an inaccessible environment. Thus, in the scenario of a single conserved quantity which is the energy, $\hat{\chi}$ may be associated with a canonical distribution, $\hat{\chi}_{\beta} = e^{-\beta \hat{H}}/\mathcal{Z}$, where $\mathcal{Z} = \text{Tr}[e^{-\beta \hat{H}}]$ is the partition function and β is set appropriately at initial times. Note that $\hat{\chi}_{\beta}$ satisfies the above constraints and we can always expect such a distribution to exist in a macroscopic system. However, since we are interested in isolated systems, we instead focus on the quantum microcanonical distribution denoted $\hat{\chi}_E$, which is the expected long time equilibrium distribution for an ensemble of isolated, macroscopic quantum systems.

The quantum microcanonical distribution [1, 29, 52] for an ensemble of identical quantum systems with energies between E and $E \pm \delta E$, is given by an equally weighted sum, $\hat{\chi}_E = \frac{1}{D} \sum_{\mu} \hat{P}_{\mu}$ where the sum is over all energy eigenstates, $\hat{P}_{\mu} = |\mu\rangle \langle \mu|$ which possess energies $\operatorname{Tr}[\hat{H} \hat{P}_{\mu}] \in [E - \delta E, E + \delta E] \forall \mu$. Here D is the number of such micro-states such that $\operatorname{Tr}[\hat{\chi}] = 1$. Further we expect for any macroscopic quantum system, such a $\hat{\chi}_E$ exists and satisfies Eq. (4), for all possible quantum states, $\hat{\rho}$, with the same energy, $\operatorname{Tr}[\hat{\rho}\hat{H}] = \operatorname{Tr}[\hat{\chi}_E \hat{H}]$.

However, we still need to determine δE , which must be derived from the initial conditions of the (isolated) system. To do this, we focus on the evolution of the variance of the energy, $V = \langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2$. Using Eq. (2), $J^{\mu\nu} = \chi_{\mu}$ and Eq. (4) we find:

$$\dot{V} = \sum_{\mu} E_{\mu}^2 \, \chi_{\mu} \, - \sum_{\nu} E_{\nu}^2 \, \rho_{\nu\nu}.$$

The above expression shows that the change in the variance of the ensemble's energy depends on the difference between the variance of the steady state, $\hat{\chi}$ and the state undergoing the evolution, $\hat{\rho}$. With the choice of the steady states, corresponding to the canonical distribution $\hat{\chi}_{\beta}$, it is seen that the variance of the energy will always evolve, for any initial (non-equilibrium) state and converge to that of $\hat{\chi}_{\beta}$. However, with the choice of $\hat{\chi}_E$, the microcanonical ensemble, if we further identify $\pm \delta E = \sqrt{V}$ as the variance in the energy of the initial state, we fix the conditions such that the average energy variance also does not change. Here, we thus make an assumption, which must ultimately be tested in experiments. We fix the model such that a macroscopic

quantum system known to be in a non-equilibrium state with energy E and variance $V = \delta E^2$, approaches at long times a microcannonical distribution $\hat{\chi}_E$, which is diagonal and uniformly distributed at all energy eigenstates lying in the microcannonical window of $[E - \delta E, E + \delta E]$. This agrees with the conventional definitions of the microcanonical ensemble [1, 29, 52].

We further note that although the choice of $\hat{\chi}_{\beta}$ does not keep energy eigenstates (except the ground state) stable under evolution, the choice of $\hat{\chi}_E$ using the above prescription keeps energy eigenstates stable. Concretely, if the initial state is a non-degenerate energy eigenstate, $|\phi\rangle = |i\rangle$, the choice of $\hat{\chi}_E = |i\rangle \langle i|$ is fixed by its zero variance and preserves the initial state under the action of Eq. (2), with the above constraints. Finally, since we expect closed systems to evolve to a microcanonical distribution, the choice of $\hat{\chi}_E$ is ultimately more preferable and ensures a consistent notion of objective quantum thermalization for isolated systems.

Note that the above procedure may be generalized to settings with multiple conserved quantities. While in the case of the canonical solutions $\hat{\chi}_{\beta}$, these would correspond to the generalized Gibbs ensemble [46], in the case of microcanonical solutions, $\hat{\chi}_E$, one simply ensures that conserved quantities remain preserved. Thus for each symmetry \hat{S} of the Hamiltonian, we must ensure that $\frac{\partial}{\partial t}\langle \hat{S}\rangle = 0$, establishing further constraints. Using Eq. (2), one scenario in which these constraints are satisfied is if \hat{S} commutes with $\hat{L}_{\mu\nu}$, showing that transitions are allowed only for micro-states within the sectors fixed by the initial value of the conserved quantity, denoted $S = \langle \hat{S} \rangle$ and its variance $\delta S^2 = \langle \hat{S}^2 \rangle - \langle \hat{S} \rangle^2$, like in the previous case. Then in this scenario, the microcanonical distribution is given by, $\hat{\chi}_{(E,S)} = \frac{1}{D} \sum_{\psi} \mathbb{P}_{\psi}$ where $\hat{\mathbb{P}}_{\psi} = |\psi\rangle \langle \psi|$ are projections onto the energy basis, and the sum is over all energy eigenstates, such that $\langle \psi | \hat{H} | \psi \rangle \in [E - \delta E, E + \delta E]$ and further, $\langle \psi | \hat{S} | \psi \rangle \in$ $[S - \delta S, S + \delta S]$ for each micro-state. As before, D is the number of such micro-states such that $Tr[\hat{\chi}] = 1$. A more detailed analysis of the constraints pertaining to multiple conserved quantities is left for the future. For our treatment, it suffices that a microcanonical $\hat{\chi}$ exists for any given (thermodynamic) system.

With these constraints satisfied, we are now in a position to write down the final expressions for the modified quantum dynamics, allowing an objective notion of thermalization, with stable end states as an appropriate quantum microcanonical distribution, $\hat{\chi}_E$ with energy as its conserved quantity. Individual macroscopic quantum systems with (initial) quantum energy expectation, $E_{\psi} = \langle \psi | \hat{H} | \psi \rangle$ and variance $V_{\psi} = \langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2$ evolve as per the following (norm preserving) quantum stochastic process:

$$d |\psi\rangle = -\frac{i}{\hbar} \hat{H} |\psi\rangle dt + \sum_{\mu,\nu} d\hat{G}^{\mu\nu}_{\chi} |\psi\rangle , \qquad (5)$$
$$d\hat{G}^{\mu\nu}_{\chi} |\psi\rangle := \sqrt{\mathcal{A}^{\mu}} \left[\hat{L}_{\mu\nu} - \langle \hat{L}_{\mu\nu} \rangle \right] |\psi\rangle dW^{\mu\nu}_{t}$$
$$+ \mathcal{A}^{\mu} \left[\langle \hat{L}^{\dagger}_{\mu\nu} \rangle \hat{L}_{\mu\nu} - \frac{1}{2} \hat{L}^{\dagger}_{\mu\nu} \hat{L}_{\mu\nu} - \frac{1}{2} \langle \hat{L}_{\mu\nu} \rangle \langle \hat{L}^{\dagger}_{\mu\nu} \rangle \right] |\psi\rangle dt.$$

Here, $\mathcal{A}^{\mu} = \left(\frac{\alpha \mathcal{N}}{\hbar}\right) \chi_{E}^{\mu}$ where $\chi_{E}^{\mu} = \langle \mu | \hat{\chi}_{E} | \mu \rangle$, which accounts for the dimensions. The energy (density) scale of the modification is denoted by α , and the factor of \mathcal{N} makes explicit the requirement that the modification's strength must scale with the size of the system. As explained above, $\hat{\chi}_{E}$ is an equally weighted (diagonal) distribution over all possible energy eigenstates with energies in the microcanonical interval, $[E_{\psi} - \sqrt{V_{\psi}}, E_{\psi} + \sqrt{V_{\psi}}]$ (set at initial times). While individual systems evolve via Eq. (5), the master equations for the evolution of ensembles is given by:

$$\hbar \frac{\partial \hat{\rho}}{\partial t} = -i \left[\hat{H}, \, \hat{\rho} \right] + \alpha \, \mathcal{N} \, \Lambda_{\chi}(\, \hat{\rho}\,), \tag{6}$$
$$\Lambda_{\chi}(\, \hat{\rho}\,) = \sum_{\mu,\nu} \, \chi^{\mu}_{E} \, \hat{L}_{\mu\nu} \, \hat{\rho} \, \hat{L}^{\dagger}_{\mu\nu} - \, \hat{\rho}.$$

The above master equations evolve density (statistical) operators via the GKSL generator, Λ_{χ} . Together, Eq. (5) and Eq. (6) realize a physically viable model of objective quantum thermalization (OQT) for isolated and macroscopic quantum systems. Since \mathcal{N} scales with the system size, the effect of the modifications is larger for more macroscopic systems, thus admitting a quantum-to-classical transition from entangled states to classical configurations [9, 10].

To show that states evolve via Eq. (6) to equilibrium, we may compute the trace distance, $D_{\chi} = \text{Tr}[(\hat{\rho} - \hat{\chi}_E)^2]$. Using Eq. (6), we find $\dot{D}_{\chi} = -\omega D_{\chi}$ (where $\omega = 2\alpha \mathcal{N}/\hbar$), showing that the trace distance exponentially decreases over time, $D_{\chi} = D_{\chi}(0) \exp(-\omega t)$, where $D_{\chi}(0)$ is the trace distance at initial times. This shows that all states $\hat{\rho}$ which are not already the steady states, approach $\hat{\chi}$. It also implies that quantum Poincaré recurrences or analogous entanglement reversal events [2, 5] do not occur for macroscopic systems undergoing the OQT dynamics. This further implies, that the (von Neumann) entropy is non-decreasing for all states which are not already at equilibrium.

Note that the stochastic nature of the dynamics of each ensemble member (Eq. (5)), itself, generates entropy for ensembles, due to the stochasticity of the individual trajectories, similar to objective collapse models [17]. This is seen readily by noting the evolution of the von Neumann entropy, $S = -\text{Tr}[\hat{\rho} \log \hat{\rho}]$, for ensembles evolving via Eq. (6). The change in the entropy is given by,

 $\dot{S} = -\text{Tr}\left[\log \hat{\rho} \frac{\partial}{\partial t} \hat{\rho}\right]$ since $\frac{\partial}{\partial t} \text{Tr} \hat{\rho} = 0$. Evaluating further using Eq. (6), we find (with $\alpha \mathcal{N}/\hbar = 1$):

$$\dot{S} = \operatorname{Tr}\left[\left(\hat{\rho} - \hat{\chi}_E\right)\log\hat{\rho}\right] = \mathbb{D}\left[\hat{\chi}_E \mid\mid \hat{\rho}\right] + \mathbb{H}_{\chi} - S.$$

Here, $\mathbb{D}[\hat{\chi} || \hat{\rho}] = \operatorname{Tr}[\hat{\chi} \log \hat{\chi}] - \operatorname{Tr}[\hat{\chi} \log \hat{\rho}]$ (if $\operatorname{supp}(\hat{\chi}) \subset$ $\operatorname{supp}(\hat{\rho})$ and $+\infty$ otherwise) is the Umegaki quantum relative entropy, which is non-negative [66, 67] and decreases under CPTP maps [66, 67] such as Eq. (6). In the above expression, $\mathbb{H}_{\chi} = -\text{Tr}[\hat{\chi}_E \log \hat{\chi}_E] = \log D$, which is the von Neumann (Shannon) entropy of the associated microcanonical distribution and D denotes its dimension. Since $\mathbb{H}_{\chi} > S$, necessarily for all non-equilibrium states, the above expression implies that we have $\dot{S} > 0$ for all non-equilibrium states and $\dot{S} \geq 0$ generically. This shows the emergence of the second law of thermodynamics and a modified quantum H-theorem, independent of considerations such as the specifics of the macroscopic system, its initial states or any agent-dependent restrictions. Since the total ensemble approaches an appropriate microcanonical distribution, $\hat{\chi}_E$, all sub-system observables also thermalize appropriately [1, 52].

Summarizing, the OQT model predicts that macroscopic quantum systems thermalize generically, at the level of the microscopic pure states and such a notion of thermalization is independent of considerations on the space of observables or states. Hence, the model in Eq. (5) and Eq. (6) showcase an objective notion of quantum thermalization, one independent of all agent-specific arguments or approximations. The OQT model also admits a notion of irreversibility for each individual system through the dynamics of Eq. (5). Since each ensemble member remains pure (Eq. (5) maps pure states to pure states), the von Neumann entropy in the individual system do not increase. For the entire ensemble, as shown above, the von Neumann entropy does increase and appropriately maximizes in each setting.

Contrary to other approaches towards resolving the QTP, in our approach, the entropy increase in ensembles is associated to the underlying stochastic dynamics of each individual system. Further, for individual systems, there is a well-defined notion of distance from any particular equilibrium measure, whether it is $\hat{\chi}_E$ above, or any other coarser measure. Thus, this framework allows us to account for the expectation that generic thermodynamic quantum systems should thermalize (at the level of ensembles), while clarifying that pure states of individual systems, at long times remain fluctuating near equilibrium phenomena in single systems—such as the ticking of clocks and active biological systems [2, 26, 33], can exist within this paradigm.

In contrast to objective collapse theories, various similarities and differences may be noted with the OQT model presented above. Firstly, the form of Eq. (5) is a stochastic Schrödinger equation similar to other collapse models, although, probability weights in the analogous collapse bases are not Martingale processes, which opens up the possibility of superluminal signalling due to the lack of Born's rules [38–40]. However, superluminal signalling is not allowed in the OQT model presented, as will be discussed shortly.

Another subtle difference is that the OQT model requires a specification of an average energy, E_{ψ} -sector, which in turn controls the structural specifications of Eq. (5) and Eq. (6). Once the E_{ψ} -sector (and variance) is specified corresponding to the energy of the initially identical ensemble members, Eq. (5) results in a scrambling dynamics for each ensemble member (controlled by the instances of the stochastic contributions), while the noise-averaged ensemble distribution approaches $\hat{\chi}_E$. In other words, the thermalization model is not apriori defined for all setups, but is obtained in each case via an analysis of the Hamiltonian, similar to the construction of a recently proposed class of objective collapse models motivated by spontaneous symmetry breaking [9, 17–19].

Note that an important shortcoming of the OQT model is that it is driven by uncorrelated white noise, which cannot constitute a physical process [9, 17, 19]. However, the model may be viewed as an effective Markovian description of a colored noise driven model of OQT. This mapping is made precise via the so called multiscale noise homogenization procedure—a temporal renormalization scheme on the space of colored noise driven models [9, 17, 19]. The key take away of such a procedure would be that multiple colored noise models would ultimately flow to Eq. (5) and Eq. (6) [9, 17, 19]. Further, the model is constructed for countable spectrum of energy eigenstates, and may be generalized to the continuum setting by employing multi-parameter Wiener processes or spacetime white noise [9, 19]. The analysis of physical consistency in such continuum and colored noise driven models is left for future studies.

Further, analogous to open problems in relativistic collapse models [68], the construction of OQT models in the relativistic regime remains another challenge and is left for future investigations. In this context, the formulation in Eq. (5) via the E_{ψ} -sectors, may be particularly advantageous towards treating quantum systems in the strict thermodynamic limit, as well as quantum field theories, where these E_{ψ} -sectors map on to disjoint super-selection sectors, wherein each such sector furnishes its own representation of the operator algebra [7].

Superluminal signalling — Having constructed a physically viable OQT model, we will now show that it does not allow faster than light communication or signalling. Unphysical scenarios allowing superluminal signalling, is a common issue encountered in modified quantum theories [38–40]. It is linked to the possibility that spatially separated parties can infer each others actions faster than light, given the modified dynamics on the entire Hilbert space. Residual non-linearites in the master equation are one key indication of such possibilities, since ensembles evolving via non-linear master equations do not remain equivalent and allow superluminal signalling [38–40]. Since the master equation in Eq. (6) is linear and of the GKSL form, it is assured that there is no superluminal signalling as long as the thermalization operators, $\hat{L}_{\mu\nu}$ have local support [40]. This is re-verified presently by an explicit calculation, following the witness of superluminal signalling established in Ref. [40].

Consider the usual setup of Alice and Bob in their spatially separated labs with a tensor product Hilbert space, $\mathcal{H}_A \otimes \mathcal{H}_B$. They have access to an ensemble of entangled particles. One may imagine that Alice additionally possesses a macroscopic quantum system (like a quantum gas in a box) and Alice's share of a high energy, entangled particle impinges on it. While Alice's system equilibrates using the modified dynamics of Eq. (5), Bob may perform local operations and measurements on his entangled particle. For an ensemble of such setups, if Bob notices any change in his measurements due to Alice's share undergoing non-trivial dynamics, Alice has effectively sent a signal to Bob faster than light. We will now see how this is disallowed given that the thermalization of Alice's system is local.

Let Alice and Bob share an ensemble of arbitrary entangled states of the form, $|\Psi_{AB}\rangle := \sum_{\mu,\sigma} \psi_{\mu,\sigma} |\mu_A\rangle \otimes$ $|\sigma_B\rangle$. While Bob has access to states, indexed by σ , $(|\sigma_B\rangle \in \mathcal{H}_B \equiv \mathbb{C}^M)$ which he can projectively measure, Alice has access to an *N*-state macroscopic system $(|\mu_A\rangle \in \mathcal{H}_A \equiv \mathbb{C}^N)$ entangled with Bob's share. Let Alice's macroscopic state now thermalize *locally* following the dynamics of Eq. (5) and Eq. (6). This implies that in particular, $\hat{G}_{\chi} \equiv \hat{G}_{\chi}^A \otimes \hat{\mathbb{I}}_B$, and the operators, $\hat{L}_{\mu\nu} \equiv \hat{L}_{\mu\nu}^A \otimes \hat{\mathbb{I}}_B$, where $|\mu_A\rangle, |\nu_A\rangle \in \mathcal{H}_A$.

Since Bob has access to local projective measurements and operators $(\hat{\mathbb{I}}_A \otimes \hat{O}_B)$, he may only access the reduced state $\hat{\rho}_B = \text{Tr}_A[\hat{\rho}_{AB}]$ [40]. Here, $\hat{\rho}_B$ is the reduced density matrix for Bob which is obtained after a partial trace $(Tr_A[.])$ procedure on the entire density matrix $\hat{\rho}_{AB}$. The change in Bob's ensemble averages (for local observables) due to Alice's system thermalizing is then given by $\frac{\partial \hat{\rho}_B}{\partial t} = Tr_A[\frac{\partial}{\partial t}\hat{\rho}_{AB}] = 0$ unconditionally, where $\hat{\rho}_{AB}$ evolves via Eq. (6) with local thermalization operators as described above. This clearly shows that a spatially extended macroscopic entangled state may thermalize locally, however superluminal signalling is prohibited within the dynamics of Eq. (5) and Eq. (6).

If $\frac{\partial \hat{\rho}_B}{\partial t} \neq 0$, typically due to non-linear expectation values in the master equation, then indeed Bob would have been able to distinguish his ensemble before and during objective thermalization, allowing Alice to effectively signal faster than light. Clearly this is not possible within the proposed model with local thermalization operators, since all non-linearities cancel given the FDR, which thus is also seen to guarantee no superluminal signalling in the

theory. With these physicality requirements satisfied, we now focus on incorporating objective collapse.

Spontaneous Universal Irreversibility—The objective quantum thermalization model (Eq. (5) and Eq. (6)) and the various objective collapse models [10, 17–20, 56] showcase the possibility of a previously unrecognized manner of universality in non-equilibrium phenomena for quantum systems approaching the thermodynamic limit [9]. In particular such models modify the deterministic and time-reversal symmetric evolution of quantum systems and allow them to undergo fundamentally stochastic and irreversible dynamics observed in nature, but disallowed in standard quantum dynamics. These modifications are extensive and their strength depends on the system size, fundamentally affecting microscopic and macroscopic quantum matter differently and are thus experimentally verifiable [56–58]. Together, a hybrid dynamics of both objective collapse and objective thermalization is seen to resolve both the quantum measurement problem and the QTP within the same theory [9].

This further draws attention to the possibility of an unknown universal mechanism which could allow the emergence of irreversibility and randomness fundamentally. Note that in this view, the standard practice of instantaneously substituting 'by hand', equilibrium states, symmetry broken states or projected states (during measurements) may be seen as a minimal, economical way of mapping onto the observed physics [9]. Indeed, since these are inherently dynamical and non-equilibrium processes, the next refinement towards describing such physics is in the Markovian approximation, where, due to the various constraints, unique equations of the form described here (Eq. (5) and Eq. (6)) in the context of thermalization, and those discussed in the context of other objective collapse models [9, 10, 19, 56, 59] are the only physically viable possibilities.

Following this line of thought, we may construct a unique class of universal models showcasing fundamental quantum stochasticity and irreversibility for systems approaching the *thermodynamic limit* [1, 7, 69] or equivalently, systems beyond the so called *Heisenberg's* cut [7, 9, 70], or indeed, systems in the quantum-toclassical crossover regime [9, 10, 56]. Such terminology, although used in different situations, may be seen to indicate the same regime where quantum theory requires modifications so as to account for the spontaneous emergence of classicality. In particular, such modifications, employing hybrid models, with generators admitting both objective collapse and objective thermalization are henceforth termed models of Spontaneous Universal Irreversibility (SUI). We will now briefly describe the construction of one such model.

We consider a recently established objective collapse model termed Spontaneous Unitarity Violations (SUV) [17–21, 71, 72]. SUV is motivated by extending spontaneous symmetry breaking [69, 73] to the non-equilibrium regime and its dynamics allow macroscopic quantum objects to spontaneously break their symmetries, disallowed in standard quantum dynamics. The same model applied to a measurement setting—composed of a system under measurement, macroscopic measurement devices and an environment—allows the entire setup to undergo quantum state reduction to classical states, where the devices break a symmetry and allow measurements to be performed [17–19], such as a pointer breaking translation symmetry.

Unlike other collapse models [10, 56], there is no preferred basis chosen apriori, instead, in each situation the SUV model uniquely determines the symmetry broken basis of the setup's Hamiltonian as its basis of wavefunction reduction [17, 19–21, 72]. Further, in the Markovian regime, the SUV model reduces to an effective continuous spontaneous localization (CSL) model, collapsing in the symmetry breaking basis [19].

Since all three stochastic and irreversible phenomena spontaneous symmetry breaking, quantum measurements and thermalization—occur only for systems approaching the thermodynamic limit [10, 69], a SUI model constructed using the SUV model [9, 17–19] and the OQT model (Eq. (5) and Eq. (6)) can function together seamlessly and yields a universally applicable theory of quantum irreversibility in the non-relativistic regime.

In particular, such a model consists of two distinct modifications, one allowing dynamical quantum state reduction and one allowing thermalization, given by the (norm-preserving) quantum stochastic process (on the entire Hilbert space \mathcal{H} of the isolated setup):

$$d |\psi\rangle = -\frac{i}{\hbar} \hat{H} |\psi\rangle dt + d\hat{G}_R |\psi\rangle + d\hat{G}_\chi |\psi\rangle. \quad (7)$$

$$d\hat{G}_R |\psi\rangle = -\frac{\mathcal{JN}}{2\hbar} \sum_k \left(\hat{\mathbb{P}}_k - \langle\hat{\mathbb{P}}_k\rangle\right)^2 |\psi\rangle dt$$

$$+ \sqrt{\frac{\mathcal{JN}}{\hbar}} \sum_k \left(\hat{\mathbb{P}}_k - \langle\hat{\mathbb{P}}_k\rangle\right) |\psi\rangle dW_t^k.$$

Here, as before, $|\psi\rangle$ denotes the state of a single isolated (thermodynamic) system with its standard Hamiltonian \hat{H} . Further, $d\hat{G}_{\chi} = \sum_{\mu\nu} d\hat{G}_{\chi}^{\mu\nu}$, which is given in Eq. (5), $d\hat{G}_{\chi}$ generates objective thermalization and allows systems to approach an appropriate notion of microcanonical equilibrium, $\hat{\chi}$, given conserved quantities. Further, here, we consider the case of dG_R being the SUV generator of wavefunction reduction in the Markovian limit [9, 17, 19]. Other generators from different objective collapse models [10, 56] may also be considered, but in all such physically viable generators, stringent constraints require that both stochastic contributions and deterministic contributions exist and are related by a fluctuation dissipation relationship [9, 17, 19], as also seen in the OQT model before. Further, just like the OQT model (Eq. (5) and Eq. (6)) which allows the equilibrium microcanonical distribution $\hat{\chi}$ to emerge at long times, a diagonal distribution corresponding to Born's rules, $\hat{\rho}_{\rm B}$, emerges from the SUV dynamics at long times (Eq. (7) with $\hat{H} = 0$ and $d\hat{G}_{\chi} = 0$).

Given a measurement setting with initial state, $|\psi\rangle = \sum \alpha_i |i\rangle$, represented in the collapse (symmetrybroken) basis, the diagonal Born distribution is $\hat{\rho}_{\rm B} = \sum_k |\alpha_k|^2 \hat{\mathbb{P}}_k$. Here the projectors $\hat{\mathbb{P}}_k = |k\rangle \langle k|$ projects onto symmetry broken states, determined from \hat{H} , which crucially allows the SUV approach to model quantum state reduction and spontaneous symmetry breaking as two descriptions of the same irreversible and inherently random phenomenon [17, 19–21, 71, 72].

The corresponding master equations for ensembles of systems evolving via Eq (7) are (linear) GKSL master equations with two contributions:

$$\hbar \frac{\partial \hat{\rho}}{\partial t} = -i \left[\hat{H}, \, \hat{\rho} \right] + \mathcal{J} \, \mathcal{N} \, \Lambda_R(\, \hat{\rho}\,) + \alpha \, \mathcal{N} \, \Lambda_\chi(\, \hat{\rho}\,). \tag{8}$$

Here, $\Lambda_R(\hat{\rho}) := \sum_k \hat{\mathbb{P}}_k \hat{\rho} \hat{\mathbb{P}}_k - \hat{\rho}$, generates reduction of the wave-function in the symmetry breaking basis, which allows an ensemble averaged description of both spontaneous symmetry breaking and quantum measurement in their appropriate settings [19]. Λ_{χ} is defined in Eq. (6) and generates objective thermalization in ensembles as discussed before. Further, both models posses norm preservation, energy conservation, and no-superluminal signalling [17, 19], strengthening their viability as physically relevant models for the dynamics of isolated systems. These properties are ultimately related to the fluctuation dissipation relationship which requires that the deterministic and stochastic contributions of the modifications must be related, which may possibly be a hint towards their underlying origin [9, 17, 19].

Together, the combined effect of Λ_R and Λ_{χ} describe competing, hybrid thermalization-reduction dynamics with long time steady, equilibrium distributions denoted by $\hat{\rho}_{\infty}$. If they exist, they are obtained by the steady state condition in each situation, given by $i\left[\hat{H}, \hat{\rho}_{\infty}\right] = \mathcal{JN} \Lambda_R(\hat{\rho}_{\infty}) + \alpha \mathcal{N} \Lambda_{\chi}(\hat{\rho}_{\infty}).$ The $\hat{\rho}_{\infty}$ interpolates between the diagonal Born distribution, $\hat{\rho}_{\mathrm{B}}$, and the microcanonical distribution, $\hat{\chi}$, allowing an unified description of quantum state reduction, spontaneous symmetry breaking and quantum thermalization, for isolated macroscopic quantum systems. Since the microcanonical distribution was identified by Maxwell, Boltzmann and Gibbs in their respective works [2, 25], we term the $\hat{\rho}_{\infty}$ as a hybrid Born-Maxwell-Boltzmann-Gibbs-microcannonical distribution. Further analysis of the steady states, towards understanding the quantum-to classical transitions in particular settings [9, 17, 26, 33] is left for future studies.

SUI models thus possess the potential of an un-ambiguous description of the quantum-to-classical crossover physics in both individual systems and ensembles. For microscopic systems the SUI models are practically indistinguishable from standard quantum theory, but due to their differing dynamics, yields falsifiable predictions for quantum systems approaching the thermodynamic limit [56–58]. Further, although our considerations in this article were restricted to non-relativistic systems, ultimately, our program aims to provide a unified description of irreversibility and stochasticity in generic space-times for generic quantum systems of particles and fields at the quantum-to-classical crossover regime. The construction of relativistic SUI models and the analysis of dedicated experimental protocols falsifying them are left for future studies.

Conclusions — The deterministic and time-reversal symmetric nature of quantum dynamics poses a foundational challenge in reconciling it with the irreversible approach to equilibrium, held as an axiom in equilibrium statistical theory. Standard approaches toward a reconciliation employ epistemic restrictions, based on what may be practically known by specific agents. However, in such approaches, an agent-independent notion of equilibrium in isolated systems, is not possible, nor is it understood how a single, isolated, macroscopic system irreversibly approaches equilibrium. Our work resolves this tension by instead advocating that quantum mechanics is an effective theory, requiring corrections to describe the dynamics of systems approaching the thermodynamic limit.

We proposed a minimal stochastic modification of quantum dynamics, such that for small systems, the modified dynamics is practically indistinguishable from Schrödinger's evolution, while macroscopic systems can approach equilibrium dynamically. This objective quantum thermalization (OQT) model ensures that macroscopic systems, governed by corrections scaling with system size, evolve irreversibly toward thermal equilibrium described by a microcanonical distribution. Crucially, this approach to equilibrium is valid for single systems as well as for all possible observables or initial states of a given (macroscopic) system, transcending agentdependent restrictions inherent in other frameworks.

We showed that a fluctuation-dissipation relation guarantees that the model is norm-preserving and avoids superluminal signalling, ensuring physical consistency. We showed that constraints towards energy conservation and equilibrium steady states may be simultaneously satisfied and constructed an OQT model with increasing von Neumann entropy in ensembles of isolated systems. The ensemble was shown to converge to the micocanonical equilibrium distribution at long times, for thermodynamic systems, independent of further considerations. Owing to the differing dynamics in macroscopic quantum systems, the OQT model presented in this article is falsifiable via dedicated protocols in the mesoscopic regime; which measure thermalization time scales, temporal correlations and their scaling with system size, in isolated quantum systems.

We also considered the integration of OQT with objective collapse theories, which yield a unified framework, termed spontaneous universal irreversibility (SUI); addressing both the quantum measurement problem and the quantum thermalization problem. We constructed a particular SUI model which describes macroscopic systems undergoing stochastic dynamics that drive spontaneous symmetry breaking, wavefunction collapse, and objective thermalization, while remaining indistinguishable from standard quantum theory for microscopic systems.

This hybrid model reconciles the emergence of classicality and equilibrium thermodynamics from its quantum mechanical constituents, resolving both the quantum measurement problem and the quantum thermalization problem, within the same theory. By ensuring norm preservation, linear master equations and energy conservation, the (non-relativistic) SUI model provides a self-consistent theory of quantum irreversibility and quantum-to-classical crossover dynamics, where the interplay of stochastic collapse and thermalizing dynamics results in emergent, long time equilibrium steady states, corresponding to a hybrid Born-Maxwell-Boltzmann-Gibbs-microcanonical distribution.

The investigation of such SUI models have deep foundational significance as well as many practical applications. In the non-relativistic regime, apart from its applications in gaseous or condensed matter systems [5], the SUI models further open up possibilities of finer analyses and characterization of noise in quantum devices, such as quantum computers, a pressing open problem [9, 74]. SUI models are also interesting to consider in the relativistic regime and in general space-times, an open problem with both conceptual and technical subtleties [68].

Spontaneous symmetry breaking [69, 73] is a corner stone of the standard model of particle physics and since the SUI model provides its natural extension to the non-equilibrium regime [17, 19–21, 71, 72], relativistic SUI models are of considerable interest towards treating quantum-to-classical transitions in the high energy setting. This is made all the more interesting due to the recent collider tests of quantum foundations [75, 76]. Indeed, relativistic SUI models could be used to investigate known conundrums of thermalization time scales in relativistic heavy ion-collision physics [77], as well as in phase transitions and non-equilibrium physics in the setting of early universe cosmology [78, 79], and in the black-hole information paradox [80].

Crucially, note that although we have constructed equations of motion admitting long time equilibrium steady states, being the microcanonical and Born distribution, our analysis does not specify *why* these are the preferred steady states distributions in the first place. Thus, a key open problem is also the source of such modifications, which would presumably inform these concerns. In the context of objective collapse models, its origins have been previously argued to emerge due to an instability of superposed space-times of massive quantum objects [81, 82].

In closing, our results challenge conventional interpretations of quantum theory and the emergence of classicality and equilibrium. By positing stochasticity in the fundamental dynamics of physical systems, our framework lays bare an understanding of quantum-toclassical transitions and the emergence of equilibrium, offering a unified dynamical framework for spontaneous symmetry breaking, quantum state reduction and equilibration. Our findings thus motivate and encourage a re-assessment of the foundations of physics via thorough theoretical and experimental exploration of potential universal mechanisms which could underlie the irreversibility and stochasticity that we observe in the universe.

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