

Linking the KM3-230213A Neutrino Event to Dark Matter Decay and Gravitational Waves Signals

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Abstract. The KM3NeT collaboration recently reported the detection of an ultra-high-energy neutrino event, dubbed KM3-230213A. This is the first observed neutrino event with energy of the order of $\mathcal{O}(100)$ PeV, the origin of which remains unclear. We interpret this high energy neutrino event results from the Dirac fermion dark matter (DM) χ decay through the right-handed (RH) neutrino portal assuming the Type-I seesaw mechanism for neutrino masses and mixings. Furthermore, dark matter χ is assumed to charged under $U(1)_X$ dark gauge symmetry, which is spontaneously broken by the vacuum expectation value of the dark Higgs Φ . In this scenario, the DM can decay into a pair of Standard Model (SM) particles for $v_\Phi \gg m_\chi$, which we assume is the case. If the DM mass is around 440 PeV with a lifetime 5×10^{29} sec, it can account for the KM3-230213A event. However, such heavy DM cannot be produced through the thermal freeze-out mechanism due to overproduction and violation of unitarity bounds. We focus on the UV freeze-in production of DM through a dimension-5 operator, which helps in producing the DM dominantly in the early Universe. We have also found a set of allowed parameter values that can correctly account for the DM relic density and decay lifetime required to explain the KM3NeT signal. Moreover, we have generated the neutrino spectra from the two-body decay using the HDMSpectra package, which requires the dark Higgs vacuum expectations value (VEV) to be much larger than the DM mass. Finally, the large value of the dark Higgs field VEV opens up the possibility of generating GW spectra from cosmic strings. We have found a reasonable set of parameter values that can address the KM3NeT signal, yield the correct value of the DM relic density through freeze-in mechanism, and allow for possible detection of GW at future detectors.

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1 Introduction

The KM3NeT collaboration recently announced the detection of a muon event with an energy of $\mathcal{O}(100 \text{ PeV})$, originating from a muon neutrino. This event, which now sets the record for the highest-energy neutrino ever detected, has been named KM3-230213A. The inferred energy of the ultra-high-energy neutrino lies in the range $72 \text{ PeV} \leq E_\nu \leq 2.6 \text{ EeV}$, with a median energy of 220 PeV. The neutrino flux required to fit the data is around [1]

$$E_\nu^2 \frac{d\Phi_\nu}{dE_\nu} = 5.8_{-3.7}^{+10.1} \times 10^{-8} \text{GeVcm}^{-2}\text{s}^{-1}\text{sr}^{-1}, \quad (1.1)$$

with $E_\nu = 72 \text{ PeV} - 2.6 \text{ EeV}$.

To produce such an ultra-high-energy neutrino via cosmic-ray interactions (such as pp , $p\pi^-$, or $p\gamma$ collisions) in typical astrophysical sources would require protons accelerated to EeV energies. However, the KM3NeT collaboration investigated several potential source candidates such as blazars and gamma-ray bursts using various electromagnetic telescope catalogs [1]. No conclusive evidence for a counterpart was found along the direction of the neutrino. To explain the event, there are a lot of hypotheses proposed in Refs. [2–23].

The spatial distribution of DM and interaction properties between DM and SM particles still remain crucial issues in both cosmology and particle physics. In particular, the nature of its interaction like gravitational or otherwise is essential for understanding its role in the formation of large scale structures and the evolution of the Universe. In this study, we consider a scenario in which dark matter is extremely heavy and have a lifetime much longer than the age of the Universe. Even though lifetime is much longer than the age of Universe, at the present Universe, DM can decay into SM particles, including active neutrinos. If DM mass is above $\mathcal{O}(100) \text{ TeV}$, then it cannot be produced via thermal freeze-out. We will consider an alternative DM production mechanism. There is one unique feature of DM explanation for the KM3NeT neutrino event. There should be sharp energy cut-off in the neutrino spectrum, which can be tested by future IceCube and KM3NeT data. In the present work, we have produced the neutrino flux from the two body decay of the DM which is larger than the three body decay for dark higgs VEV larger than the DM mass. We have used HDMSpectra package [24] for producing the neutrino and photon fluxes. With the 440 PeV DM mass and $5 \times 10^{29} \text{ sec}$ lifetime, we can successfully explain the KM3NeT data, whereas the photon flux obtained from the same decay is lower than the LHAASO–KM2A and EAS–MSU data.

In the present work, the $U(1)_X$ dark gauge symmetry breaks spontaneously at a very high scale nonzero VEV of dark Higgs field Φ . Then the cosmic string networks can form during the dark gauge symmetry breaking. The cosmic string production in the early Universe as a remnant of the symmetry-breaking phase transition has been predicted in a wide variety of Grand Unified Theories (GUT) as well [25–27]. The $U(1)$ gauge symmetry spontaneously breaks down, resulting in a non-trivial value for the first homotopy group; hence, cosmic strings form during the symmetry breaking. Once the cosmic string networks are formed, they achieve a scaling regime and produce loops that emit gravitational waves (GW) and vanish over time.

Cosmic strings produced from the spontaneous symmetry breaking of the $U(1)$ gauge symmetry have a string tension related to the VEV of the BSM scalar as $\mu \sim v_\phi^2$ [28]. In the scaling regime, the fraction of cosmic string networks energy budget compared to the total energy budget is proportional to string tension μ . Therefore, a very high value of the $U(1)$ -breaking scalar VEV can be probed at different proposed detectors, namely Square Kilometre Array (SKA) [29], Laser Interferometer Space Antenna (LISA) [30], Big Bang Observer (BBO)/DECi hertz Interferometer Gravitational wave Observatory (DECIGO) [31], Einstein Telescope (ET) [32, 33], Cosmic Explorer (CE) [34], and LIGO/Virgo [35, 36]. As we will see, $G\mu > 2 \times 10^{-11}$ ($v_\phi > 10^{14}$ GeV)¹ is already in tension with the European Pulsar Timing Array (EPTA) data [37]. Moreover, we will see that $G\mu < 10^{-19}$ is beyond the reach of the proposed GW detectors.

In this work, we propose a local $U(1)_X$ gauge symmetry with a SM-singlet heavy Dirac fermion χ that is charged under $U(1)_X$ with the RH neutrino portal. Dark $U(1)_X$ is to be spontaneously broken by the nonzero VEV of dark Higgs field Φ . Then DM χ will decay into the SM particles through the RH neutrino portal. We consider a heavy Dirac fermion χ as a DM candidate, whose very late decay into SM particles is responsible for the KM3-230213A event. In this model, we also include super-heavy right-handed (RH) neutrinos. Such RH neutrinos appear in the type-I seesaw mechanism, which explains the origin of light neutrino masses. It is worth mentioning that among the three RH neutrinos, two account for the light neutrino masses, while the remaining one assists in the long-lived DM decay and its production through an effective operator generated by super-heavy RH neutrino. Since a very high value of the VEV v_ϕ is required in enhancing the two-body decay of DM over the three-body decay, which demands $v_\phi \gg M_\chi$, where M_χ is the DM mass, typically in the $\mathcal{O}(100)$ PeV scale for explaining the KM3NeT signal. Such a heavy VEV leads to detection prospects of the present scenario at future GW detectors. Moreover, we have also produced such heavy DM candidates by the freeze-in mechanism through an effective higher-dimensional operator. We have found that such a high-mass regime can be easily achieved for such high values of the VEV without fine-tuning the model parameters. In other words, we can explain the DM relic density and lifetime using the model parameters described in Sec. 2, with $T_R \sim 10^{10}$ GeV, $\kappa \sim 10^{-4}$, $M_N \sim 10^{12}$ GeV, and $y \sim 10^{-22}$. The parameters value are natural for the freeze-in kind of DM production, and a tiny value of y is needed to ensure a DM lifetime of $\mathcal{O}(10^{29})$ sec, which is typical for all DM scenarios requiring a decaying DM particle.

This paper is organized as follows. In Sec. 2 we introduce our DM model with dark $U(1)_X$ gauge symmetry, heavy right-handed neutrino portal and Higgs portal interactions. In Sec. 3 we outline the general formalism for calculating neutrino flux from galactic and extragalactic DM decay. We present both total and differential decay widths for the relevant

¹ G is the gravitational constant.

three-body decay in our model and compare the numerical results with IceCube data. In Sec. 5 we discuss a possible mechanism to generate the correct relic density for DM within our scenario of decaying heavy Dirac fermion DM and detection constraints. In Sec. 4, we discuss gravitational signature induced by cosmic strings. Finally, we conclude in Sec. 6.

2 Dark $U(1)_X$ model

In this section, we recapitulate the decaying heavy Dirac fermion DM model proposed in Ref. [38]. Let us start with dark $U(1)_X$ gauge symmetry including a Dirac fermion DM χ and a dark Higgs field Φ [38]. Their charge assignments under the dark $U(1)_X$ symmetry are as follows.: $(Q_\chi, Q_\Phi) = (1, 1)$. We consider the renormalizable and gauge-invariant Lagrangian that includes singlet right-handed (RH) neutrinos N 's which are gauge singlets:

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} + \frac{1}{2} \bar{N} i \not{\partial} N - \left(\frac{1}{2} m_N \bar{N}^c N + y \bar{L} \tilde{H} N + \text{h.c.} \right) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} \sin \epsilon X_{\mu\nu} F_Y^{\mu\nu} \\ & + D_\mu \Phi^\dagger D^\mu \Phi - V(\Phi, H) + \bar{\chi} (i \not{D} - m_\chi) \chi - (\kappa \bar{\chi} \Phi N + \text{h.c.}), \end{aligned} \quad (2.1)$$

where $L = (\nu, l)^T$ denotes the left-handed (LH) SM lepton doublet, and H is the SM Higgs doublet. The field strength tensor of the $U(1)_X$ gauge field X_μ is given by $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, while $F_Y^{\mu\nu}$ corresponds to that of the SM gauge field for $U(1)_Y$ hypercharge. The parameter ϵ represents the kinetic mixing between the SM hypercharge and the new $U(1)_X$ gauge boson. The covariant derivative is defined as $D_\mu = \partial_\mu - ig_X Q_X X_\mu$. Here we introduce two new types of Yukawa couplings, y and κ , which are assumed to be real for simplicity.

The scalar potential $V(\Phi, H)$, including the dark Higgs field Φ , is given by

$$V(\Phi, H) = \lambda_H \left(H^\dagger H - \frac{v_H^2}{2} \right)^2 + \lambda_{\phi H} \left(H^\dagger H - \frac{v_H^2}{2} \right) \left(\Phi^\dagger \Phi - \frac{v_\phi^2}{2} \right) + \lambda_\phi \left(\Phi^\dagger \Phi - \frac{v_\phi^2}{2} \right)^2. \quad (2.2)$$

Both electroweak (EW) and dark gauge symmetries are spontaneously broken by the nonzero vacuum expectation values of H and Φ . In the unitarity gauge, the scalar fields read

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H + h \end{pmatrix} \quad \text{and} \quad \Phi = \frac{v_\phi + \phi}{\sqrt{2}}. \quad (2.3)$$

Two electrically neutral scalars h and ϕ can mix with each other due to the Higgs-portal coupling, $\lambda_{\phi H}$. Thanks to this mixing, dark Higgs ϕ can decay into SM particles.

Likewise, three neutral gauge bosons, photon A_μ , Z_μ and X_μ can mix with each other due to the kinetic mixing ϵ . Here we take the kinetic mixing to be small, $\epsilon \ll 1$. New physical gauge boson Z'_μ in the mass eigenstate is mostly dark photon X_μ which can decay SM fermion pairs.

When the RH neutrino N_R is significantly heavier than the dark matter χ , it can be integrated out, yielding the dim-5 effective operator [38]:

$$\frac{y\kappa}{m_N} \bar{\chi} \Phi H^\dagger L + \text{h.c.} \quad (2.4)$$

This term enables the decay of χ by allowing it to couple to light particles. However, dark matter can remain long-lived due to the superheavy RH neutrino masses and the appropriate choice of Yukawa coupling constants. After the spontaneous breaking of gauge symmetries, the

operator in Eq. (2.4) leads to the emergence of various higher-dimensional effective operators [38]:

$$\frac{y\kappa}{2} \frac{v_\phi v_H}{m_N} \bar{\chi}\nu, \quad \frac{y\kappa}{2} \frac{v_\phi}{m_N} \bar{\chi}h\nu, \quad \frac{y\kappa}{2} \frac{v_H}{m_N} \bar{\chi}\phi\nu, \quad \frac{y\kappa}{2} \frac{1}{m_N} \bar{\chi}\phi h\nu. \quad (2.5)$$

If kinematically allowed, all of the above operators induce χ decays into various channels with fixed relative branching ratios. Under the assumption that χ is much heavier than ϕ , Z' , h , Z , and W , the mass operator $\bar{\chi}\nu$ in Eq. (2.5) gives rise to a suppressed mixing between DM and the active neutrino. The mixing angle θ between DM and active neutrino is approximately given by

$$\theta \simeq \frac{y\kappa}{2} \frac{v_\phi v_H}{m_N m_\chi}. \quad (2.6)$$

Consequently, the gauge interactions of χ and ν induce decay modes for χ as follows:

$$\chi \rightarrow Z'\nu, Z\nu, W^\mp l^\pm, \quad (2.7)$$

with their branching ratios being proportional to $\sim v_H^2 : v_\phi^2 : 2v_\phi^2$. The dim-4 operators $\bar{\chi}h\nu$ and $\bar{\chi}\phi\nu$ in Eq. (2.5) induce the following decays of χ :

$$\chi \rightarrow h\nu, \phi\nu, \quad (2.8)$$

with their branching ratios being proportional to $\sim v_\phi^2 : v_H^2$. Therefore, we can evaluate all the decay branching ratios in this model.

Additionally, this model permits the three-body decay $\chi \rightarrow \phi h\nu$, which can dominate depending on the mass hierarchy between χ and v_ϕ . We can compare two body decay channel to $\chi \rightarrow \phi h\nu$ three body decay channel:

$$\frac{\Gamma_2(\chi \rightarrow h\nu, \phi\nu)}{\Gamma_3(\chi \rightarrow \phi h\nu)} \simeq 16\pi^2 \frac{v_\phi^2 + v_H^2}{m_\chi^2}. \quad (2.9)$$

There are another three-body decay channels:

$$\chi \rightarrow \phi/Z' + h + \nu, \quad \phi/Z' + Z + \nu, \quad \phi/Z' + W^\pm + l^\mp,$$

with branching ratios 1 : 1 : 2 because of the Goldstone boson equivalence theorem.

In determining the neutrino and photon fluxes, we have used the HDMspectrum package [24], which provides accurate SM particle ($\bar{p}, e^\pm, \gamma, \nu$ etc.) spectra when dealing with the heavy particles decaying or pair-annihilating promptly into a pair of SM particles. In particular, we have focused on two-body decays of DM into SM particles. When using the HDMspectrum package, it is more challenging to determine the spectra when DM decays into beyond SM particles (such as dark photon or dark Higgs) or DM undergoes three-body decays, as this requires knowledge of the branching ratios and the energies of the final-state particles, which can vary in the case of three-body decays. A dominant two-body decay into SM particles can be ensured by choosing $v_\phi \gg m_\chi$ [see Eq.(2.9)]. Most interestingly, this choice of the large v_ϕ opens up a new possibility to detect GWs from cosmic string networks with the future GW detectors in the present work. Therefore, the most important DM decay channels we shall consider are $\chi \rightarrow \nu h, \nu Z, \ell^\pm W^\mp$ where all the final states involve the SM particles, and not dark sector particles.

3 Neutrino Flux from DM decay

The neutrino flux from dark matter decay is split into two parts: galactic and extragalactic contributions. Galactic neutrino flux at kinetic energy E from DM decay in our Milky Way dark halo is given by

$$\left. \frac{d\Phi_\nu^G}{dE_\nu d\Omega} \right|_{E_\nu=E} = \frac{D_g}{4\pi M_\chi} \sum_i \Gamma_i \left. \frac{dN_\nu^i}{dE_\nu} \right|_{E_\nu=E}, \quad (3.1)$$

where Γ_i is partial width for decay channel i , dN_ν^i/dE_ν is the neutrino spectrum at production obtained by HDMSpectra package [24]. D_g is the D-factor from our galaxy which is defined by

$$D_g = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} d\Omega \int_0^{r_{\max}} dr' \rho(r'), \quad (3.2)$$

where $\Delta\Omega$ is the angular region of detection, $r' = \sqrt{r_\odot^2 + r^2 - 2r_\odot r \cos\psi}$, r is the distance to earth from the DM decay point, $r_\odot \simeq 8.5$ kpc is the distance between galactic center and the Solar system. For the galactic DM density distribution, we use the following standard NFW profile [39],

$$\rho_\chi(r) = \frac{\rho_0}{r/r_c(1+r/r_c)^2}, \quad (3.3)$$

with parameters $r_c \simeq 20$ kpc and ρ_0 is determined by $\rho_\chi(r_\odot) = 0.3$ GeV/cm³. For KM3-230213A event, the neutrino flux is not sensitive to the choice of DM density profile in our galaxy because a neutrino does not travel from the galactic center. When DM comes from the galactic center, the value of the integration in Eq.(3.2) is different depending on which DM profile we used.

We can also get the extragalactic or cosmic contribution, by taking cosmic expansion into account, namely the redshift effect [40]:

$$\left. \frac{d\Phi_\nu^{EG}}{dE_\nu d\Omega} \right|_{E_\nu=E} = \int_0^\infty dz \frac{1}{\mathcal{H}(z)(1+z)} \left(\frac{1+z_0}{1+z} \right)^3 \frac{\bar{\rho}_\chi(z)}{m_\chi} \sum_i \Gamma_i \left. \frac{dN_\nu^i}{dE'_\nu} \right|_{E'_\nu=(1+z)E} e^{-s_\nu(E_\nu, z)}, \quad (3.4)$$

where the average cosmological DM density is $\bar{\rho}_\chi(z) = \rho_\chi^0(1+z)^3$, the factor $\left(\frac{1+z_0}{1+z}\right)^3$ incorporates the effect of cosmological dimming of surface brightness due to the expansion-induced dilution of the source, and the factor $(\mathcal{H}(z)(1+z))^{-1}$ converts the redshift interval into a corresponding proper distance interval. Hubble parameter is given by,

$$\mathcal{H}(z) = \mathcal{H}_0 \sqrt{\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4}, \quad (3.5)$$

where \mathcal{H}_0 is the present-day value of the Hubble parameter. Since we observe the neutrino at $z_0 = 0$, the differential flux of extragalactic neutrino reads

$$\left. \frac{d\Phi_\nu^{EG}}{dE_\nu d\Omega} \right|_{E_\nu=E} = D_{\text{eg}} \int_0^\infty dz \frac{1}{1+z \sqrt{\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4}} \left. \frac{dN_\nu^i}{dE'_\nu} \right|_{E'_\nu=(1+z)E} e^{-s_\nu(E_\nu, z)}, \quad (3.6)$$

where E' is red-shifted to E as $E' = (1+z)E$. D_{eg} is the D-factor from extragalaxy which is defined as

$$D_{\text{eg}} = \frac{\Omega_\chi \rho_c}{4\pi M_\chi \tau_\chi \mathcal{H}_0} = 1.6 \times 10^{-16} \left(\frac{100 \text{PeV}}{M_\chi} \right) \left(\frac{10^{29} \text{s}}{\tau_\chi} \right) \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}, \quad (3.7)$$

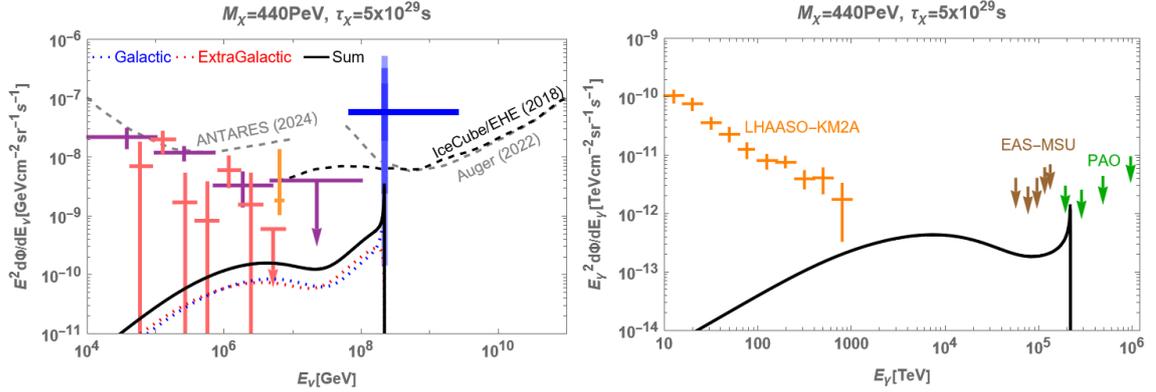


Figure 1. Neutrino (left-panel) and Gamma-ray (right-panel) spectra from DM χ decay with $M_\chi = 440\text{PeV}$ and lifetime $\tau_\chi = 1/\Gamma = 5 \times 10^{29}\text{s}$. In the left panel, bounds come from IceCube [43, 44]. Blue cross corresponds to KM3NeT with 3σ C.L [1]. It presents the galactic (blue dotted curve) and extragalactic (red dotted curve) neutrino flux. In the right panel, orange crosses correspond to gamma-ray constraints from LHAASO-KM2A [45] whereas EAS-MSU [46] and PAO [47] limits are shown in brown and green arrows, respectively.

where the critical energy density $\rho_c = \frac{3H_0^2}{8\pi G} = 5.5 \times 10^{-6}\text{GeV}/\text{cm}^3$ and $\Omega_\chi \simeq 0.27$ is DM χ 's fraction, Ω_Λ , Ω_m and Ω_r are energy fractions of dark energy, total matter, and radiations, respectively. We have adopted the Planck results [41] for the numerical evaluation. Lastly, $s_\nu(E_\nu, z)$ is the neutrino opacity of the Universe, assuming that the neutrinos are massless. The neutrino opacity $s_\nu(E_\nu, z)$ is given by [42]

$$s_\nu(E_\nu, z) = \begin{cases} 7.4 \times 10^{-17}(1+z)^{7/2}(E_\nu/\text{TeV}), & \text{for } 1 \ll z < z_{\text{eq}} \\ 1.7 \times 10^{-14}(1+z)^3(E_\nu/\text{TeV}), & \text{for } z \gg z_{\text{eq}} \end{cases} \quad (3.8)$$

where $z_{\text{eq}} \sim 3200$ [41] is the redshift value corresponds to the matter-radiation equality epoch during the Universe evolution.

Now we discuss neutrino and gamma-ray fluxes induced by decaying heavy DM. In the left panel (LP) of Fig. 1, we show the neutrino flux originated from both our galaxy, $(\ell, b) = (210.06^\circ, -11.13^\circ)$, and extragalaxy. The neutrino flavor ratio arriving at the Earth is assumed to be $1 : 1 : 1$. Here we consider the arrival direction (RA: 94.3° , Dec: -7.8°) of KM3-230213A with an angular uncertainty of $\pm 1.5^\circ$ with 1σ C.L [1]. In the left panel, we describe the neutrino spectra from the heavy DM decay. Red crosses denote the IceCube data [43, 44]. Left dashed line represents the bound coming from the ANTARES data [48]. Right dashed lines above $\sim 10\text{PeV}$ are upper limits of neutrino flux coming from no-observations of neutrinos beyond that in IceCube and Auger [43, 44]. The orange cross is the most energetic neutrino event detected by IceCube neutrino telescope [49]. We also show the preferred flux of the KM3-230213A event by Blue cross with 3σ C.L. Blue and red dotted lines show galactic and extragalactic contributions from heavy decaying DM, respectively. We can explain the neutrino event detected by the KM3-NeT neutrino telescope through the decaying heavy DM.

On the other hand, heavy DM χ decay can generate secondary gamma-ray flux. In the right panel (RP) of Fig. 1, we show the gamma-ray spectra induced by the heavy DM decay, including photons from the cascade decays. We compute the gamma-ray spectra from the inner Galactic plane, $15^\circ < \ell < 125^\circ$, $-5^\circ < b < 5^\circ$. The orange crosses correspond to LHAASO-KM2A data [45]. Upper limites from EAS-MSU [46] and PAO [47] Collaborations

are shown in brown and green colors, respectively. We note that the photon flux predicted in our model is below the upper bounds from LHAASO-KM2A and EAS-MSU data.

4 Gravitational waves from cosmic string

In this section, we discuss GW production from cosmic strings, which are formed due to the $U(1)$ gauge symmetry breaking in the early Universe. After their formation, the cosmic string network enters the scaling regime, where the strings achieve a balance between the slow redshift of the horizon-length strings due to the $a(t)^{-2}$ factor in density dilution during cosmic expansion and the transfer of energy from long strings to the production of closed string loops [50]. The velocity-dependent one-scale (VOS) model provides a good analytical description of long strings by taking into account their characteristic length and mean string velocity. A detailed description of the VOS model is given in Refs. [51–53].

During the evolution of cosmic strings, large loops are produced that predominantly emit GWs, while highly boosted small loops primarily lose energy through simple redshifting [54, 55]. The long loops lose energy in the form of GWs at a constant rate governed by the following equation,

$$\frac{dE}{dt} = -\Gamma G\mu^2 \quad (4.1)$$

where $\Gamma = 50$ is obtained from simulations [56–58], G is the gravitational constant, and μ is the string tension. Therefore, loops with an initial size $l_i = \alpha t_i$, where t_i is the loop formation time and $\alpha = 0.1$ is an approximate loop size parameter [58, 59], shrink with time, and their length $l(t)$ at time t can be expressed as,

$$l(t) = \alpha t_i - \Gamma G\mu(t - t_i). \quad (4.2)$$

The total energy loss from an individual loop also depends on the mode frequency $f_k = \frac{2k}{l}$ ($k = 1, 2, 3 \dots$ is the mode number) during its normal mode oscillation. As found in Refs. [58, 59], the emission rate per mode scales with k as $k^{-4/3}$ and takes the form,

$$\Gamma^{(k)} = \frac{k^{-4/3}\Gamma}{\sum_{m=1}^{\infty} m^{-4/3}} \quad (4.3)$$

where $\sum_{m=1}^{\infty} m^{-4/3} \simeq 3.60$. The frequency \tilde{f} emitted at time \tilde{t} can be related to the frequency f at time t by accounting the redshift effect as follows,

$$f = \left[\frac{a(\tilde{t})}{a(t)} \right] \tilde{f}. \quad (4.4)$$

We can express the relic GW background from the cosmic string network in terms of GW energy density ρ_{GW} and frequency f as follows,

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \ln f}. \quad (4.5)$$

Summing over all k -modes during string oscillations, the relic GW density for frequency f can be expressed as,

$$\Omega_{GW}(f) = \sum_k \Omega_{GW}^{(k)}(f) \quad (4.6)$$

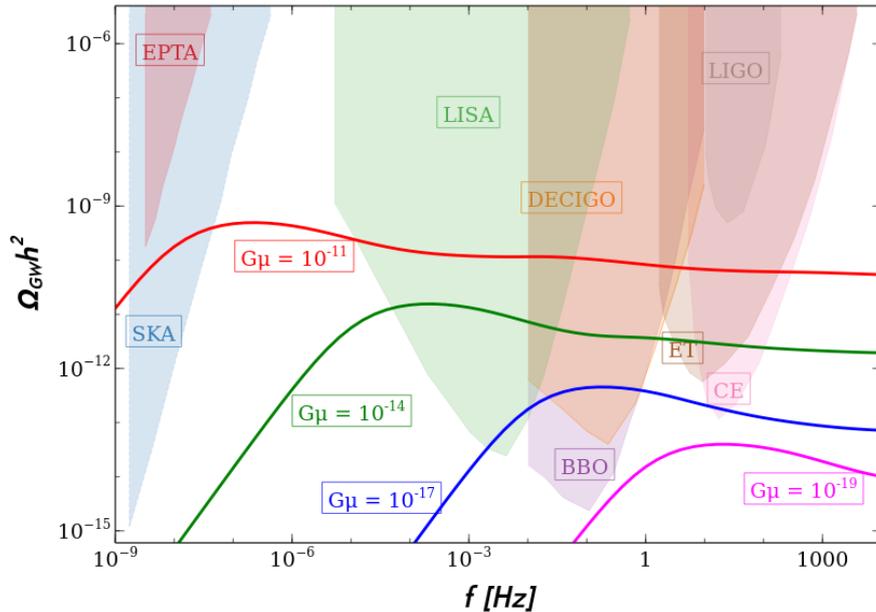


Figure 2. Variation of relic GW density with frequency for different values of string tension. Different colours represent the sensitivity prospects of various future GW detectors. EPTA data excludes cosmic string tensions $G\mu > 2 \times 10^{-11}$.

where the contribution to the GW from an individual frequency mode is given by [60],

$$\Omega_{\text{GW}}^{(k)} = \frac{1}{\rho_c} \frac{2k}{f} \frac{F_\alpha \Gamma^{(k)} G\mu^2}{\alpha(\alpha + \Gamma G\mu)} \int_{t_F}^{t_0} dt \frac{C_{\text{eff}}(t_i^{(k)})}{(t_i^{(k)})^4} \left[\frac{a(\tilde{t})}{a(t_0)} \right]^5 \left[\frac{a(t_i^{(k)})}{a(\tilde{t})} \right]^3 \theta(t_i^{(k)} - t_F). \quad (4.7)$$

In the above expression, $F_\alpha = 0.1$ denotes the fraction of energy released by long strings into loops. The parameter t_F is the time when the string network reaches the scaling regime, which occurs shortly after cosmic string formation during symmetry breaking. The quantity t_0 denotes the present time, and \tilde{t} is the time of GW emission, over which the integration runs from t_F to t_0 . The function $C_{\text{eff}}(t_i^{(k)})$ takes the value 5.5 during the radiation-dominated era and 0.41 during the matter-dominated era in the early Universe, as obtained from simulation [58, 59, 61, 62]. The loop formation time contributing to mode k is given by

$$t_i^{(k)}(\tilde{t}, f) = \frac{1}{\alpha + \Gamma G\mu} \left[\frac{2k}{f} \frac{a(\tilde{t})}{a(t_0)} + \Gamma G\mu \tilde{t} \right]. \quad (4.8)$$

In determining the relic GW spectra with the frequency as given in Eq. 4.7, we have used `micrOMEGAs` [63] for computing the scale factor $a(t)$ which depends on the relativistic *d.o.f* of the universe as well as we have used the inbuilt integration routine in `micrOMEGAs` for computing the integration.

In Fig. 2, we show the prospects for GW produced from cosmic strings, as detectable by various proposed future detectors. In the plot, the low-frequency GW regime primarily originates from the matter-dominated era, while the high-frequency regime arises from the radiation-dominated era [60]. The string tension $G\mu > 2 \times 10^{-11}$ is already in tension with the EPTA data [37], so we consider $G\mu < 10^{-11}$ in our analysis. This upper limit is already more stringent than the bound from the CMB, which is $G\mu < 2 \times 10^{-7}$ [64]. As shown, different

values of $G\mu$ will be probed by different proposed experiments. For example, $G\mu = 10^{-11}$ can be explored by all the proposed detectors, namely SKA [29], LISA [30], BBO/DECIGO [31], ET [32, 33], and CE [34]. On the other hand, $G\mu = 10^{-17}$ is beyond the reach of SKA but may be detectable by DECIGO and BBO, while values of $G\mu < 10^{-19}$ fall below the sensitivity limits of all future detectors. The range of $G\mu$ accessible to GW detectors corresponds to a BSM scalar VEV in the range $v_\phi = 10^{10}$ GeV to 10^{14} GeV, which is also crucial for the KM3NeT signal, particularly in the scenario where dark matter decays are dominated by two-body standard model particles in the final states, as discussed earlier. Moreover, this range of v_ϕ can also yield the correct relic abundance of dark matter for $\mathcal{O}(100)$ PeV-scale DM masses through freeze-in mechanism, as will be discussed in more detail in Sec. 5. This provides an intriguing possibility of correlating freeze-in dark matter production with observable signals at GW detectors.

5 Dark Mater Production

In this section, we study the relic density estimation of the heavy DM using the freeze-in mechanism from ultraviolet dependent physics. For $\mathcal{O}(100)$ PeV scale DM, the thermal freeze-out mechanism fails to generate the correct relic density due to the violation of unitarity bounds [65]. Therefore, we consider an alternative production mechanism, known as freeze-in [66]. In our model, the right-handed (RH) neutrino is very heavy and beyond the relevant scale of our study, so it can be integrated out safely. As a result, the interactions between the DM and other particles are mediated via dimension-5 operators. We consider a UV freeze-in mechanism for DM production where DM relic density depends on the UV physics *e.g.* reheating temperature T_R . The squared amplitude for the DM production process $\phi\phi \rightarrow \bar{\chi}\chi$ is given by:

$$|\mathcal{M}|_{\phi\phi \rightarrow \bar{\chi}\chi}^2 = 2 \left(\frac{2\kappa^2}{M_N} \right)^2 (s - 4M_\chi^2) \quad (5.1)$$

and the corresponding cross section is:

$$\sigma_{\phi\phi \rightarrow \bar{\chi}\chi} = \frac{1}{8\pi s} \left(\frac{2\kappa^2}{M_N} \right)^2 \left[\frac{s - 4M_\chi^2}{s - 4M_\phi^2} \right]^{1/2} (s - 4M_\chi^2) \quad (5.2)$$

where we assume $M_N \gg M_\chi, M_\phi$. DM production is dominated at very high temperatures, $T \gg M_\chi, M_\phi$, allowing us to safely neglect the masses of the particles compared to the center-of-mass energy \sqrt{s} . Under this approximation, the cross section simplifies to:

$$\sigma_{\phi\phi \rightarrow \bar{\chi}\chi} = \frac{1}{8\pi} \left(\frac{2\kappa^2}{M_N} \right)^2. \quad (5.3)$$

The Boltzmann equation governing DM production from pair or $\phi\phi$ annihilation is:

$$\frac{dY_\chi}{dz} = \frac{2M_{sc}s(T)}{z^2\mathcal{H}(T)T} \left[\langle\sigma v\rangle_{\phi\phi \rightarrow \bar{\chi}\chi} Y_\phi^2 - \langle\sigma v\rangle_{\bar{\chi}\chi \rightarrow \phi\phi} Y_\chi^2 \right], \quad (5.4)$$

where $z = M_{sc}/T$ and M_{sc} is a free mass scale which can be taken as DM mass. The Hubble parameter is given by $\mathcal{H}(T) = 1.66\sqrt{g_\rho(T)}\frac{T^2}{M_{\text{Pl}}}$, and the entropy density is $s(T) = \frac{2\pi^2}{45}g_s(T)T^3$,

where $g_\rho(T)$ and $g_s(T)$ are the effective relativistic degrees of freedom of the Universe corresponding to the energy density and entropy density, respectively. We assume that DM is produced via freeze-in from ultraviolet physics, with an initially negligible abundance. Hence, $Y_\chi = 0$ at early times, becoming nonzero once production starts from the thermal bath. The thermally averaged cross section times velocity takes the form [67],

$$\langle\sigma v\rangle_{\phi\phi\rightarrow\chi\bar{\chi}} = \frac{g_\phi^2 T}{2(2\pi)^4 n_\phi^2} \int \sigma_{\phi\phi\rightarrow\chi\bar{\chi}}(s - 4M_\phi^2) \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right). \quad (5.5)$$

Since DM production occurs at very high temperatures, we can again neglect initial and final state masses. The thermal average then becomes:

$$\langle\sigma v\rangle_{\phi\phi\rightarrow\chi\bar{\chi}} \simeq \frac{g_\phi^2 T}{2(2\pi)^4 n_\phi^2} \frac{1}{8\pi} \left(\frac{2\kappa^2}{M_N}\right)^2 \int_0^\infty s^{3/2} K_1\left(\frac{\sqrt{s}}{T}\right). \quad (5.6)$$

Using the above expression and $z = M_{sc}/T$, the Boltzmann equation can be rewritten as:

$$\frac{dY_\chi}{dT} = -\frac{T^5}{\mathcal{H}(T)s(T)(4\pi^5)} \left(\frac{2\kappa^2}{M_N}\right)^2. \quad (5.7)$$

Substituting expressions for $H(T)$ and $s(T)$, and integrating from the reheating temperature T_R down to the present-day temperature T_0 , the comoving number density is:

$$Y_\chi \simeq \frac{3.6 M_{\text{pl}}(T_R - T_0)}{\pi^7 g_s \sqrt{g_\rho}} \left(\frac{2\kappa^2}{M_N}\right)^2, \quad (5.8)$$

Once we calculate the co-moving number density, the resulting DM relic density is then given by [68],

$$\Omega_\chi h^2 = 2.755 \times 10^{14} \left(\frac{M_\chi}{\text{PeV}}\right) Y_\chi. \quad (5.9)$$

We vary the model parameters over the following ranges:

$$10^{-7} \leq \kappa \leq 1, \quad 10^{-20} \leq G\mu \leq 10^{-10}, \quad 10^{10} \leq M_N, [\text{GeV}] \leq 10^{16}, \quad 10^{-30} \leq y \leq 10^{-20}, \\ 1.7 \times 10^8 \leq M_\chi, [\text{GeV}] \leq 5.5 \times 10^9, \quad 10^{10} \leq T_R, [\text{GeV}] \leq 10^{13}. \quad (5.10)$$

We consider a DM mass range that can explain the KM3NeT events and apply the decay lifetime constraint $3.6 \times 10^{29} \leq \frac{\tau_\chi}{f_\chi} \leq 6.3 \times 10^{29}$, where τ_χ is the DM lifetime and f_χ is its fractional contribution to the total DM abundance (we consider 1–100%). In scenarios where DM is a subdominant component, a multi-component DM framework may be invoked, as explored in the literature [69]. As shown above, the right-handed neutrino mass M_N is always greater than the reheating temperature T_R , meaning N_R is not thermally populated in the early Universe. Usually, freeze-in DM cannot be probed by ongoing direct, indirect detection, or collider searches. However, thanks to the KM3-230213A event, we might have a hint for decaying heavy DM produced via freeze-in process. In our model, we can find solution for both DM relic density and KM3-230213A event without taking unnatural values for the relevant parameters in the model. Also, gravitational wave is generated in the early

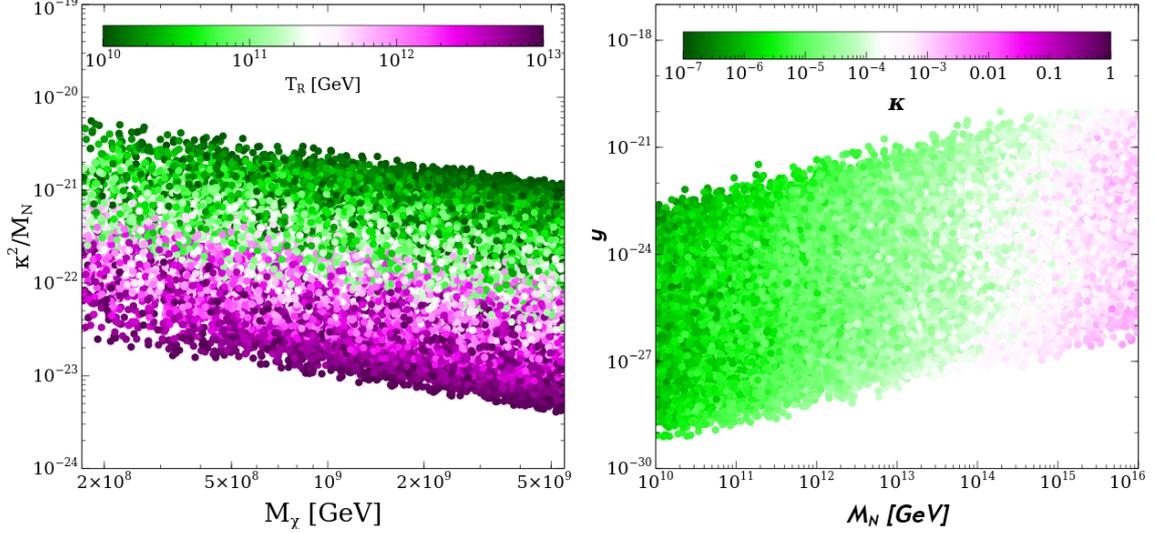


Figure 3. In the left panel (LP), we show the scatter plot in the $(M_\chi, \frac{\kappa^2}{M_N})$ plane, whereas the right panel (RP) displays the scatter plot in the (M_N, y) plane. The color gradient in the LP represents different values of the reheating temperature T_R , while in the RP, it corresponds to different values of the coupling κ . The other parameters, which are not shown, have been varied as listed in Eq.(5.10).

Universe due to large Dark Higgs VEV. It has detection prospects at future gravitational wave detectors.

In the LP of Fig. 3, we show the scatter plot in the $(M_\chi, \frac{\kappa^2}{M_N})$ plane after imposing the requirement that the DM mass and its fractional contribution lie within the (1–100)% range, providing the correct value of $\frac{T_\chi}{f_\chi}$ necessary to explain the KM3NeT signal. An anti-correlation between M_χ and $\frac{\kappa^2}{M_N}$ is observed, which can be understood from Eqs. (5.8) and (5.9). These equations show that the DM relic density is proportional to $M_\chi T_R \left(\frac{2\kappa^2}{M_N}\right)^2$. Therefore, to achieve a fixed relic density, an increase in one parameter must be compensated by a decrease in the other, leading to the observed anti-correlation. From the color bar, we also observe that as $\frac{\kappa^2}{M_N}$ increases, lower values of T_R are required (as seen along the y-axis). Similarly, along the x-axis, as M_χ increases, the required T_R also decreases to maintain the correct relic density. In the RP, we present the scatter plot in the (M_N, y) plane, with the color bar representing different values of κ . The parameters in this plot are related to both the DM relic density and decay width. As M_N increases, the relic density tends to decrease, while it increases with larger κ . On the other hand, the decay width increases with increasing y . To obtain the correct value of $\frac{1}{f_\chi \Gamma_\chi}$ for explaining the KM3NeT signal, we observe a correlated variation between M_N and y : an increase in M_N lowers f_χ , while an increase in y raises Γ_χ , keeping the product $f_\chi \Gamma_\chi$ approximately constant. Furthermore, to obtain the correct DM relic density, M_N and κ must also vary in a correlated (roughly linear) manner, as indicated by the color gradient.

In the LP and RP of Fig. 4, we show the allowed regions in the $(G\mu, y)$ and (f_χ, τ_χ) planes, respectively. The color variation in the LP indicates different values of the DM lifetime τ_χ , while in the RP, it corresponds to the values of κ^2/M_N . In the LP, we observe an anti-correlation between $G\mu = (v_\phi/M_{\text{pl}})^2$ and the Yukawa coupling y . This is because the decay width is proportional to $(yv_\phi)^2$; thus, for larger values of $G\mu$, a smaller value of y is required

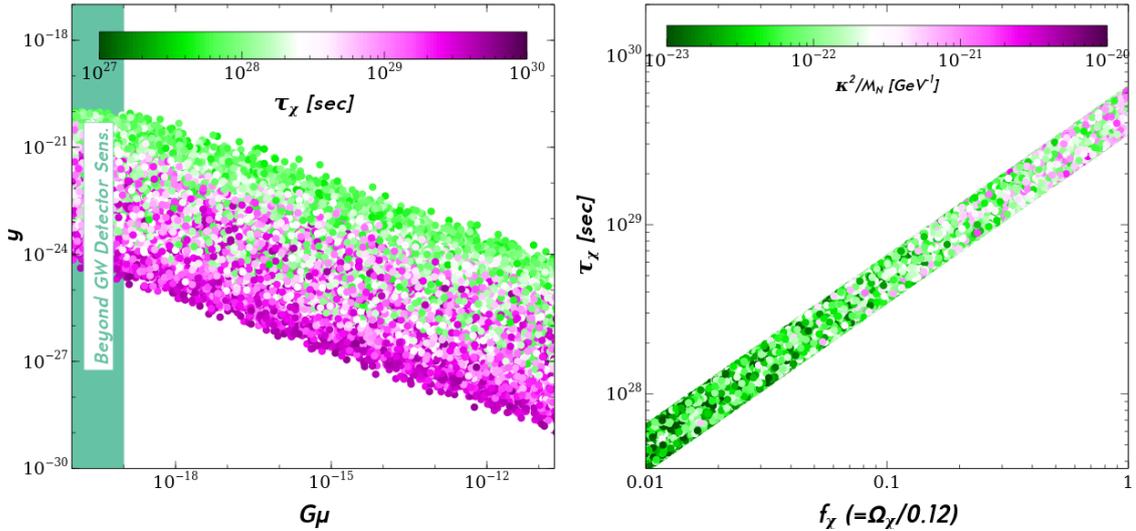


Figure 4. The LP and RP show scatter plots in the $(G\mu, y)$ and (f_χ, τ_χ) planes, respectively. In the LP, the color gradient represents different values of τ_χ , while in the RP, it corresponds to values of κ^2/M_N .

to achieve the correct decay width that explains the KM3NeT signal. Consequently, since the DM lifetime $\tau_\chi = 1/\Gamma_\chi \propto 1/(yv_\phi)^2$, we find that τ_χ decreases with increasing $G\mu$ and also with increasing y , consistent with the color gradient in the plot. The cyan region in the LP corresponds to $G\mu < 10^{-19}$, which lies below the sensitivity of future GW detectors such as BBO and hence cannot be probed. However, the other parameter values fall within the reach of proposed GW detectors, as discussed in Sec. 4. In the RP, we display the scatter plot in the (f_χ, τ_χ) plane, where a sharp correlation emerges, reflecting the requirement to explain the KM3NeT signal. The DM relic density scales as $\Omega_\chi h^2 \propto (\kappa^2/M_N^2)^2$, while the DM lifetime follows $\tau_\chi \propto 1/(\kappa/M_N^2)$. Since both relic density and lifetime also depend on other parameters such as the DM mass M_χ , Dirac Yukawa coupling y , and the VEV v_ϕ , we observe a mixed color distribution—e.g., combinations of green and magenta points—within the same region of parameter space. As mentioned earlier, all the points shown in Figs. 3 and 4 are consistent with the KM3NeT signal, satisfy the allowed relic density range, and lie within the reach of future gravitational wave detectors.

6 Conclusion

In this work, we explore dark $U(1)_X$ gauge symmetry. This DM model can resolve the KM3-230213A event in terms of DM decay. Thanks to the heavy RH neutrino portal interaction, we can naturally obtain very tiny couplings between DM and SM particles. This heavy RH neutrino can induce DM decays into SM particles, including active neutrinos. In the case of $v_\phi \gg m_\chi$, the dominant DM decay channels are $\chi \rightarrow \nu h, \nu Z, \ell^\pm W^\mp$ but for the opposite regime $v_\phi \ll m_\chi$ DM dominantly decays to three body which case has not been explored in the present study. To explain the KM3NeT signal, we have considered DM mass at $m_\chi = 440$ PeV and its lifetime $\tau_\chi = 5 \times 10^{29}$ sec as the benchmark point. In calculating the neutrino flux, we have considered both galactic and extragalactic contributions in determining the neutrino flux and found the peak at the neutrino energy $E_\nu = 220$ PeV which can explain the KM3-230213A

data. In determining the extragalactic contribution we have taken into account the redshift effect due to the matter dilution, dimming of sources and proper length. Moreover, we have also considered neutrino opacity in determining the extragalactic contribution which reduced the flux strength by 20%. Additionally, we have also estimated the photon flux predicted by our work which are below the LHAASO-KM2A and EAS-MSU data. In determining the neutrino flux as well as the photon flux, we have used the HDMspectrum package.

As said we have focussed on $v_\phi \gg m_\chi$ for the two body dominant decay which results in the cosmic string production in the early in the detectable range. We have found that cosmic strings tension in the range $G\mu = 10^{-11}$ to 10^{-19} (corresponds to $v_\phi \sim 10^{14}$ GeV to 10^8 GeV) can be detected at the different proposed experiments like SKA, BBA/DECIGO, LISA, ET and CE. The same set of $G\mu$ range can predict the DM mass and lifetime in the correct range which can explain the KM3NeT data. In estimating the GW spectra in particular the scale factor and numerical integration we have used the popular package micrOMEGAs. Finally, we have discussed in detail the PeV-scale DM production using the UV freeze-in mechanism. We have $\phi\phi \rightarrow \bar{\chi}\chi$ process, suppressed by the heavy right-handed neutrino mass, which is dimension-5 operator and produces DM in the early Universe dominantly. We have found that $T_R \sim 10^{10}$ GeV, $\kappa \sim 10^{-4}$, $M_N \sim 10^{12}$, $v_\phi = 10^{13}$ GeV and $y \sim 10^{-20}$, we can produce the DM in the correct range and the low value of y is needed to make the DM decay lifetime $\mathcal{O}(10^{29})$ sec. We have shown a few scatter plots which can predict DM in the (1 – 100)% range and at the same time can explain the KM3NeT signal and future possibility to detect at different GW detectors.

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