

# Leveraging differentiable programming in the inverse problem of neutron stars

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Neutron stars (NSs) probe the high-density regime of the nuclear equation of state (EOS). However, inferring the EOS from observations of NSs is a computationally challenging task. In this work, we efficiently solve this inverse problem by leveraging differential programming in two ways. First, we enable full Bayesian inference in under one hour of wall time on a GPU by using gradient-based samplers, without requiring pre-trained machine learning emulators. Moreover, we demonstrate efficient scaling to high-dimensional parameter spaces. Second, we introduce a novel gradient-based optimization scheme that recovers the EOS of a given NS mass-radius curve. We demonstrate how our framework can reveal consistencies or tensions between nuclear physics and astrophysics. First, we show how the breakdown density of a metamodel description of the EOS can be determined from NS observations. Second, we demonstrate how degeneracies in EOS modeling using nuclear empirical parameters can influence the inverse problem during gradient-based optimization. Looking ahead, our approach opens up new theoretical studies of the relation between NS properties and the EOS, while effectively tackling the data analysis challenges brought by future detectors.

## I. INTRODUCTION

The nuclear equation of state (EOS) remains uncertain and an open problem in physics [1–4]. Nuclear experiments only constrain its lower density behavior, i.e., around the nuclear saturation density,  $\rho_{\text{sat}} \equiv 2.7 \times 10^{14} \text{ g cm}^{-3}$  (or equivalently, a baryon number density of  $n_{\text{sat}} \equiv 0.16 \text{ fm}^{-3}$ ). For instance, experiments measuring the neutron-skin thickness of atomic nuclei constrain the symmetry energy at nuclear saturation [5–10], while heavy-ion collisions uniquely probe isospin-symmetric matter above the nuclear saturation density [11–14]. In addition, nuclear theory calculations provide constraints on isospin-asymmetric matter up to two times the nuclear saturation density [15–17].

Matter at the most extreme densities, up to  $6–8n_{\text{sat}}$ , is only realized inside neutron stars (NSs), the remnants of core-collapse supernovae [18]. Their macroscopic properties, such as mass, radius, quadrupole moment, and tidal deformability, are determined through the Tolman–Oppenheimer–Volkoff (TOV) equations, which require the EOS for closure. Consequently, measurements of NS properties can then be used to infer the EOS [19], which is referred to as the inverse problem of NSs. Several such measurements are now available. For instance, precise radio timing of binary pulsar systems determines the masses of NSs [20, 21]. Furthermore, pulse-profile

modeling of X-ray emissions from hotspots of millisecond pulsars provides joint estimates on the mass and radius of NSs [22], and has been achieved with the Neutron star Interior Composition ExploreR (NICER) [23–29]. Additionally, the tidal deformability of NSs leaves a measurable imprint on gravitational waves (GWs) emitted during the inspiral and merger of binary neutron star (BNS) systems. This way, GW170817 [30, 31], the first BNS merger observed by Advanced LIGO [32] and Advanced Virgo [33], provided the first estimates on NS radii derived from GWs [34].

Constraining the EOS with input from nuclear physics and astrophysics is commonly done with Bayesian inference [13, 35–60] (see Ref. [61] for a recently compiled overview of EOS constraints). However, current inference methods remain computationally expensive, particularly when directly sampling a set of EOS parameters, as this requires solving the TOV equations on the fly for millions of samples. This computational burden will hinder our ability to extract precise EOS constraints from NS data delivered by future experiments, such as upcoming X-ray telescopes (e.g., eXTP and STROBE-X [62–64]) and the next-generation of ground-based GW detectors (i.e., the Einstein Telescope [65–68] and Cosmic Explorer [69]). For example, next-generation GW observatories are projected to detect approximately  $\mathcal{O}(10^5)$  BNS mergers annually [66, 70]. These experiments will measure NS radii potentially with an unprecedented accuracy of 10 – 100 meters [71–75]. This increase in both the quantity and quality of observations highlights the need for more efficient inference methods for the inverse problem of NSs.

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Ideally, such methods should be capable of solving the TOV equations on the fly to ensure accurate and robust EOS constraints.

One possible solution, thoroughly explored in past works, is to resort to machine-learning surrogates. For instance, previous works have accelerated the TOV solver by replacing it with machine learning-based approximations using regression methods [76, 77], neural networks [78–84], support vector machines [78], Gaussian processes and reduced basis methods [81], and dynamic mode decomposition [85] (see Ref. [86] for a review). Other works, rather than relying on traditional sampling techniques, use machine learning models to directly recover the EOS from NS data, e.g., with neural networks [87–102], normalizing flows [103–105], transformers [106], principal component analysis [107], and symbolic regression [108, 109]. However, training these surrogate models can take on the order of hours [110] to weeks [105] (depending on the model complexity and hardware used for training) and requires additional modeling efforts. Moreover, we are often interested in comparing inferences with different parameterizations or prior assumptions. Since emulators must be retrained whenever these change, their computational speed-up comes at the cost of reduced flexibility in inference.

In this work, we propose an alternative solution to efficiently solve the inverse problem of NSs without resorting to machine-learning emulators. In particular, we leverage differential programming and hardware acceleration with graphical processing units (GPUs) to infer the EOS from NS properties in two ways. First, we make Bayesian inference of the EOS from NS data in high-dimensional EOS parameter spaces computationally efficient. Second, we introduce a gradient-based optimization scheme that can recover the parameters of the EOS underpinning a given NS mass-radius curve. While the former method can take observational uncertainties into account, the latter can be used to improve our understanding of how accurate knowledge of NSs propagates into constraints on the EOS.

This paper is organized as follows. In Sec. II, we provide an overview of the EOS parametrization used in this work, and discuss Bayesian inference and our gradient-based optimization scheme. In Sec. III, we validate the results obtained with our Bayesian inference code and investigate its scaling as a function of the dimensionality of the EOS parameter space. Moreover, we demonstrate how our framework can deliver insights into the inverse mapping from NS data to EOS parameters. Finally, we discuss our results in Sec. IV and conclude in Sec. V.

## II. METHODS

### A. Equation of state

Several approaches exist to parametrize the EOS without resorting to a specific physical description of supra-

nuclear matter. These include piecewise polytropes [111–115], the spectral representation [116–120], a speed-of-sound parametrization [121, 122], and non-parametric models based on Gaussian processes [123–125] or neural networks [126]. In this work, we adopt the parametrization of Ref. [61].

At the lowest densities, up to  $0.5n_{\text{sat}}$ , the crust EOS is fixed to the model from Ref. [127]. At densities just above the crust-core transition, we assume that matter consists solely of nucleonic degrees of freedom in  $\beta$ -equilibrium and use a metamodel (MM) parametrization of the EOS [128–130]. This parametrizes the energy per nucleon

$$e(n, \delta) = e_{\text{sat}}(n) + e_{\text{sym}}(n)\delta^2 + \mathcal{O}(\delta^4), \quad (1)$$

where  $n$  represents the baryon number density and  $\delta = (n_n - n_p)/n$  is the asymmetry parameter,  $n_n$  and  $n_p$  denote the neutron number density and the proton number density, respectively. The isoscalar (saturation) energy  $e_{\text{sat}}$  and the isovector (symmetry) energy  $e_{\text{sym}}$  are defined as a series expansion in  $x = (n - n_{\text{sat}})/3n_{\text{sat}}$ . The Taylor expansion of the former is given by

$$e_{\text{sat}}(n) = E_{\text{sat}} + \frac{1}{2}K_{\text{sat}}x^2 + \frac{1}{3!}Q_{\text{sat}}x^3 + \frac{1}{4!}Z_{\text{sat}}x^4 + \dots, \quad (2)$$

Throughout this work, we fix  $E_{\text{sat}} = -16 \text{ MeV}$  and  $n_{\text{sat}} = 0.16 \text{ fm}^{-3}$ . The expansion of the symmetry energy is given by

$$\begin{aligned} e_{\text{sym}}(n) = & E_{\text{sym}} + L_{\text{sym}}x + \frac{1}{2}K_{\text{sym}}x^2 \\ & + \frac{1}{3!}Q_{\text{sym}}x^3 + \frac{1}{4!}Z_{\text{sym}}x^4 + \dots. \end{aligned} \quad (3)$$

The expansion coefficients in Eqs. (2) and (3) are referred to as the nuclear empirical parameters (NEPs). The MM parametrization allows us to incorporate constraints from nuclear-physics experiments or compare them with similar constraints obtained from astrophysical observations. For instance, the slope of the symmetry energy  $L_{\text{sym}}$  is correlated with both the neutron-skin thickness as well as the radii of NSs [131–133].

However, this parametrization, describing nucleonic degrees of freedom, is inadequate to model the higher-density regime probed by NSs, as new degrees of freedom might become relevant [134–136], possibly with a phase transition [137–143]. Therefore, we use a phenomenological and model-agnostic approach at higher densities. Specifically, we use a speed-of-sound extension scheme (CSE) parametrization above a breakdown density  $n_{\text{break}}$ , which is varied freely in our parametrization within  $1 - 2 n_{\text{sat}}$  [121], unless stated otherwise. At densities above  $n_{\text{break}}$ , we parametrize the speed-of-sound  $c_s$  using a non-uniform grid of points in the  $(n, c_s^2)$ -plane and construct the full speed-of-sound profile with linear interpolation. We extend this CSE to  $25n_{\text{sat}}$ , the endpoint of our EOSs, for which  $c_s^2$  is also sampled. The

speed-of-sound curve from the CSE parametrization is then matched to that of the MM parametrization with cubic splines. The EOS can then be derived from the speed-of-sound curve, see Ref. [144].

We refer to the parameters of our parametrization, encompassing the NEPs and the CSE grid points, as  $\boldsymbol{\theta}_{\text{EOS}}$ .

## B. Neutron star observables

The macroscopic properties of non-spinning, isolated, spherically symmetric NSs are determined through the TOV equations [145, 146], the general-relativistic equations determining hydrostatic equilibrium<sup>1</sup>:

$$\frac{dp}{dr} = -\frac{\varepsilon(r)m(r)}{r^2} \left[ 1 + \frac{p(r)}{\varepsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 p(r)}{m(r)} \right], \quad (4)$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon(r), \quad (5)$$

where  $\varepsilon$  is the energy density,  $r$  is the radial coordinate from the center of the NS and  $m(r)$  is the mass contained in a sphere of radius  $r$ . Here, we employ units where  $G = c = 1$ . The system of equations is closed by the EOS, which links pressure and energy density. Given a central pressure  $p_c$  of the NS, the TOV equations can be integrated outwards, with boundary conditions  $m(r=0) = 0$  and  $p(r=0) = p_c$ , until we reach the boundary of the star where  $p(r=R) = 0$ . This gives the star's radius  $R$  and gravitational mass  $M \equiv m(R)$ . Repeating this procedure for various  $p_c$  provides the mass-radius relation of NSs for that EOS.

When an NS is placed in a static, external quadrupolar tidal field  $\mathcal{E}_{ij}$ , it develops a quadrupole moment  $Q_{ij}$  [148–150]. This induced deformation is described by the  $\ell = 2$  eigenfunctions of the oscillations and, to linear order, takes the form  $Q_{ij} = -\lambda \mathcal{E}_{ij}$ , where  $\lambda$  is related to the second tidal Love number  $k_2$  through  $k_2 = \frac{3}{2}\lambda R^{-5}$ . We refer readers to Refs. [148–150] for further details. These tidal deformations occur, for instance, in BNS systems, where the gravitational field of each NS induces the tidal effects deforming its companion. This leaves an imprint on the phase of the GWs emitted during inspiral. In particular, the GW emission depends, at leading order, on each star's dimensionless tidal deformability parameter  $\Lambda$ , which is defined as

$$\Lambda = \frac{2}{3}k_2 \frac{R^5}{M^5}. \quad (6)$$

## C. Differentiable programming

Differentiable programming is a programming paradigm that enables the end-to-end differentiation of computer programs [151–153]. The derivative is computed with automatic differentiation, which exploits the fact that all numerical calculations are composed of elementary operations. The derivatives of these operations are accumulated during runtime to efficiently evaluate the gradient of a program.

As a result, the parameters used in a program can easily be optimized using, for instance, gradient descent. If the program expresses a function  $\mathcal{L}(\boldsymbol{\theta})$ , then gradient descent iteratively adapts the parameters  $\boldsymbol{\theta}$  along the gradient of the objective function:

$$\boldsymbol{\theta}^{(i+1)} \leftarrow \boldsymbol{\theta}^{(i)} \pm \gamma \nabla \mathcal{L}(\boldsymbol{\theta}^{(i)}), \quad (7)$$

where the hyperparameter  $\gamma$  is often referred to as the learning rate and the plus sign (minus sign) is used when maximizing (minimizing) the objective function  $\mathcal{L}$ .

In this work, we use JAX [154, 155], which provides the aforementioned automatic differentiation capabilities in Python. Moreover, JAX can compile Python code and execute it on hardware accelerators, such as a GPU. To exploit these features, we have developed a Python package implementing JAX-based EOS code and a TOV solver, called JESTER.<sup>2</sup> With JESTER, we can efficiently evaluate the derivative of any numerical function that depends on the EOS parameters, even if it involves solving the TOV equations to obtain the macroscopic properties of the NS as an intermediate step. In the following sections, we introduce how we leverage automatic differentiation in gradient-based samplers and a novel gradient-based inversion scheme for NSs.

## D. Bayesian inference

The inverse problem of NSs (i.e., inferring the EOS from NS data) is often tackled with Bayesian inference, where the goal is to infer the posterior distribution  $P(\boldsymbol{\theta}_{\text{EOS}}|d)$  of the EOS parameters given a dataset  $d$ , representing either NS observations or constraints from nuclear physics. This is achieved by using Bayes' theorem

$$P(\boldsymbol{\theta}_{\text{EOS}}|d) = \frac{P(d|\boldsymbol{\theta}_{\text{EOS}})P(\boldsymbol{\theta}_{\text{EOS}})}{P(d)}, \quad (8)$$

where  $P(d|\boldsymbol{\theta}_{\text{EOS}})$  is the likelihood function,  $P(\boldsymbol{\theta}_{\text{EOS}})$  the prior distribution and  $P(d)$  the Bayesian evidence.

Since the posterior distribution is in most practical applications analytically intractable, one has to sample it numerically with methods such as nested sampling [156]

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<sup>1</sup>We refer readers to Ref. [147] for a didactic derivation of the TOV equations.

<sup>2</sup><https://github.com/nuclear-multimessenger-astronomy/jester>

or Markov chain Monte Carlo (MCMC) [157]. In this work, we use FLOWMC [158, 159], which evolves Markov chains along the gradient of the likelihood toward high-likelihood regions [160]. Moreover, to improve the sampling efficiency, normalizing flows (NFs) are used as proposal distributions. NFs are a class of deep-learning models that offer tractable representations of complex probability distributions [161, 162]. During sampling, the NF is trained on-the-fly from the available MCMC samples, after which it is used as global proposal distribution, improving the mixing of the chains.

### E. Equation of state constraints

In this work, we consider constraints on the EOS from chiral effective field theory ( $\chi$ EFT), mass measurements of heavy pulsars, mass-radius constraints provided by the NICER experiment, and the tidal deformabilities measured in GW170817. Below, we provide more details on each of these constraints.

#### 1. Chiral effective field theory

At low energies,  $\chi$ EFT provides an expansion of the nuclear Hamiltonian in terms of the nucleon momenta over a breakdown scale [163, 164]. After truncating the expansion at a certain order, the nuclear many-body problem involving the Hamiltonian can be solved numerically, from which the EOS follows. The truncation introduces theoretical uncertainty, which, however, can be systematically estimated [16, 17, 165, 166]. As a result,  $\chi$ EFT provides an uncertainty band for the EOS, bounded by two pressure curves  $p_-(n)$  and  $p_+(n)$ . We constrain the EOS using this band, following an approach similar to Ref. [61]. In particular, the calculation done by Ref. [61] yields a score function  $f(p, n)$  that quantifies the consistency between a given EOS and the  $\chi$ EFT prediction. Given this score function, the likelihood is given by

$$P(\boldsymbol{\theta}_{\text{EOS}}|d_{\chi\text{EFT}}) \propto \exp\left(\int_{0.75n_{\text{sat}}}^{n_{\text{break}}} \frac{\log f(p(\boldsymbol{\theta}_{\text{EOS}}; n), n)}{n_{\text{break}} - 0.75n_{\text{sat}}}\right). \quad (9)$$

We utilize the pressure band from Ref. [15], which extended calculations for pure neutron matter to  $\beta$ -equilibrium, making it applicable to NS EOS.

#### 2. Mass measurements of pulsars

Mass measurements of the heaviest known pulsars, obtained by evaluating relativistic effects in radio timing, provide a lower bound for the maximal mass of an NS, also referred to as the TOV mass  $M_{\text{TOV}}$ . Here, we consider three massive pulsars: PSR J1614-2230 [21, 167] with a mass of  $1.937 \pm 0.014 M_{\odot}$ , PSR J0348+0432 with a

mass of  $2.01 \pm 0.04 M_{\odot}$  [168],<sup>3</sup> and PSR J0740+6620 with a mass of  $2.08 \pm 0.07 M_{\odot}$  (but see also Refs. [170, 171]). All values are quoted at the 68% credible level. Radio timing measurements  $d_{\text{radio}}$  give a posterior on the pulsar mass  $P(M|d_{\text{radio}})$ , which is well approximated by a normal distribution with the mean and standard deviation values quoted above. For each pulsar, the likelihood on the EOS parameters is then

$$P(\boldsymbol{\theta}_{\text{EOS}}|d_{\text{radio}}) \propto \frac{1}{M_{\text{TOV}}} \int_0^{M_{\text{TOV}}} P(M|d_{\text{radio}}) dM, \quad (10)$$

where the EOS dependence is implicit in  $M_{\text{TOV}}$  and we assumed a uniform distribution on the masses.

### 3. NICER

Pulse profile modeling of NICER data has provided mass-radius posteriors for various pulsars [26–29, 172, 173]. In this work, we limit ourselves to those of PSR J0030+0451 and PSR J0740+6620. These analyses have later been refined with more accumulated data and advanced models for the hotspots and instrument response [174–176]. We use posterior samples on the masses and radii from the public data releases [177–180]. Their distributions  $P(M, R|d_{\text{NICER}})$  are approximated by a Gaussian kernel density estimate. In our inference, we sample the mass  $M$  of each pulsar from a uniform prior distribution and compute the likelihood as

$$P(\boldsymbol{\theta}|d_{\text{NICER}}) \propto P(M, R(\boldsymbol{\theta}_{\text{EOS}}; M))|d_{\text{NICER}}, \quad (11)$$

where the parameters  $\boldsymbol{\theta}$  consist of the EOS parameters and the pulsar masses.

### 4. GW170817

Finally, we consider constraints from tidal deformability measurements of GW170817.<sup>4</sup> For this, one first has to determine the posterior distributions on the source parameters  $\boldsymbol{\theta}_{\text{GW}}$  of GW170817 [182–184]. Here, we take the posterior samples produced by the analysis of Ref. [185], which obtained the posterior using JIM [185–187], a GW inference toolkit implemented in JAX able to perform fast Bayesian inference on GW signals. The samples are publicly available at Ref. [188].

From the posterior on the detector-frame chirp mass, the mass ratio, and the luminosity distance, we infer the

<sup>3</sup>Note that recent work [169] argues the mass to be  $\sim 1.8 M_{\odot}$ . However, in this work, we stick to the mass estimate determined previously, to allow for a direct comparison to Ref. [61].

<sup>4</sup>The second observed BNS candidate so far, GW190425 [181], does not significantly constrain the EOS due to its lower signal-to-noise ratio and larger component masses, and therefore will not be considered in this work.

source-frame component masses  $M_i$  using the linear Hubble law, where  $i = 1, 2$  labels the two components of the binary. For the inference on the EOS, we then consider the marginalized posterior distribution on  $M_i$  and the tidal deformabilities  $\Lambda_i$ . Contrary to the case of the NICER mass-radius posteriors, a Gaussian kernel density estimate provides a poor approximation of this distribution. Therefore, we train<sup>5</sup> an NF to obtain a tractable and more accurate approximation of the marginal posterior density, which we denote by  $P_{\text{NF}}(M_i, \Lambda_i | d_{\text{GW}})$ . During inference, we sample the binary component masses  $M_i$  and use the likelihood function

$$P(\boldsymbol{\theta} | d_{\text{GW}}) \propto P_{\text{NF}}(M_i, \Lambda_i(\boldsymbol{\theta}_{\text{EOS}}; M_i) | d_{\text{GW}}), \quad (12)$$

where  $\boldsymbol{\theta}$  consists of the EOS parameters and source-frame component masses of GW170817.

#### F. Gradient-based inversion

To assess how detailed knowledge on NS properties propagates to the EOS and the parametrization we employ for it, we have to invert a complete mass-radius relation  $R(M)$  or mass-tidal deformability relation  $\Lambda(M)$ . Here, we show that such an inversion can be achieved by employing a gradient-based optimization approach for the EOS parameters, as an alternative to Bayesian inference, as the latter can be quite computationally expensive.

In practice, we consider  $N = 500$  masses  $M_i$ , evenly spaced in the range  $[1.0 M_{\odot}, M_{\text{TOV}}]$ , and the corresponding radii  $\hat{R}_i$  and tidal deformabilities  $\hat{\Lambda}_i$  of the “target” EOS, denoted by a hat. We have found that the results of the recovery do not significantly depend on the value of  $N$  and the spacing of the masses. The lower bound on the mass is conservatively chosen below the lowest reliably observed NS mass to date [190].<sup>6</sup> We then measure the distance between a sample  $\boldsymbol{\theta}_{\text{EOS}}$ , with corresponding values for the tidal deformabilities  $\Lambda_i(\boldsymbol{\theta}_{\text{EOS}})$  at  $M_i$ , and the target NS values with the following loss function:

$$\mathcal{L}(\boldsymbol{\theta}_{\text{EOS}}) = \frac{1}{N} \sum_{i=1}^N \left| \frac{\Lambda_i(\boldsymbol{\theta}_{\text{EOS}}) - \hat{\Lambda}_i}{\hat{\Lambda}_i} \right|. \quad (13)$$

By starting from a randomized initial position in the EOS parameter space and iteratively performing gradient descent on this loss function, we aim to recover a set of EOS parameters that matches the given mass-tidal-deformability relation. In particular, we use the Adam

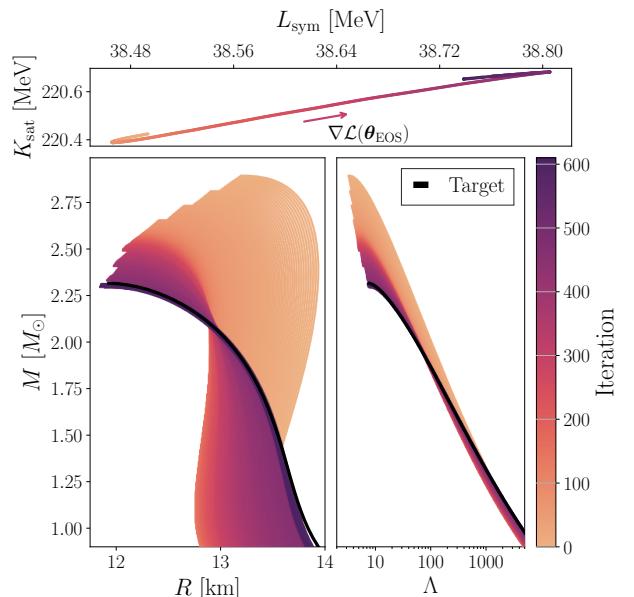


FIG. 1: Illustration of the gradient-based inversion scheme recovering a given mass-radius relation of NSs. The color gradient shows the different iterations while performing gradient descent on the objective function of Eq. (13). The gradient is computed with respect to the EOS parameters throughout the TOV equations using automatic differentiation. The top panel illustrates the trajectory of the EOS parameters, showing only  $L_{\text{sym}}$  and  $K_{\text{sat}}$  for simplicity.

optimizer, which stabilizes the descent with adaptive estimates of the first and second moments of the gradients [192]. Figure 1 illustrates our optimization algorithm. The NS curve of the EOS at the final iteration deviates from the target NS family with an error less than 100 meters in radius and less than 10 in tidal deformability for the mass range considered during optimization.

While preparing our manuscript, Ref. [193] also introduced a recovery algorithm based on optimization. However, their method uses a spectral expansion [194] and relatively low-dimensional EOS parameter spaces. Our scheme, instead, scales well to high-dimensional parameter spaces by employing gradient descent. Indeed, the illustration shown in Fig. 1 performed the inversion on the complete EOS parametrization described in Sec. II A, using eight grid points for the CSE, and was able to recover the EOS. Since this parametrization involves 26 parameters in total, this example demonstrates the applicability of our method in high-dimensional EOS spaces.

<sup>5</sup>Training this NF takes less than  $\lesssim 15$  minutes on an NVIDIA A100 GPU using FLOWJAX [189].

<sup>6</sup>Note that an NS candidate with a mass of  $\sim 0.77 M_{\odot}$  was proposed in Ref. [191], though.

TABLE I: Table of EOS parameters and their uniform prior distribution ranges, separated in the metamodel and speed-of-sound extension (CSE) parts, with the index  $i$  labeling the grid points. Throughout this work, we fix  $n_{\text{sat}} = 0.16 \text{ fm}^{-3}$  and  $E_{\text{sat}} = -16 \text{ MeV}$ .

	Parameter	Prior
Metamodel	$K_{\text{sat}}$ [MeV]	$U(150, 300)$
	$Q_{\text{sat}}$ [MeV]	$U(-500, 1100)$
	$Z_{\text{sat}}$ [MeV]	$U(-2500, 1500)$
	$E_{\text{sym}}$ [MeV]	$U(28, 45)$
	$L_{\text{sym}}$ [MeV]	$U(10, 200)$
	$K_{\text{sym}}$ [MeV]	$U(-300, 100)$
	$Q_{\text{sym}}$ [MeV]	$U(-800, 800)$
	$Z_{\text{sym}}$ [MeV]	$U(-2500, 1500)$
CSE	$n_{\text{break}}$ [ $n_{\text{sat}}$ ]	$U(1, 2)$
	$n^{(i)}$ [ $n_{\text{sat}}$ ]	$U(n_{\text{break}}, 25)$
	$(c_s^{(i)} / c)^2$	$U(0, 1)$

### III. RESULTS

#### A. Validation and computational scaling

We first demonstrate that our Bayesian inference methods reproduce previous results, and then investigate the scaling of the methodology as a function of the number of dimensions employed in the parametrization.

In order to validate our method, we compare our results to those of Ref. [61] which used an identical EOS parametrization.<sup>7</sup> Moreover, we use the same prior distribution, given in Table I. We take eight grid points for the CSE parametrization.

In Table II, we compare the 95% credible intervals of the inferred radius of a  $1.4M_{\odot}$  NS,  $R_{1.4}$ . The first rows of the table correspond to inferences employing the constraints discussed in Sec. II E individually. The bottom row shows the result when considering all constraints jointly in a single Bayesian inference run. The table demonstrates that our results qualitatively agree with Ref. [61]. However, the present work directly samples the EOS parameters, whereas Ref. [61] generated a fixed set of EOS candidates which were weighted using the constraints. This difference in sampling strategy can result in quantitative differences for the inferred  $R_{1.4}$ , as our approach allows for a higher effective sample size and more thorough exploration of the parameter space. Nevertheless, for all constraints considered here, we do observe quantitative agreement between both approaches for  $M_{\text{TOV}}$ , the pressure of the EOS at  $3n_{\text{sat}}$ , and the

TABLE II: Comparison of inferred  $R_{1.4}$  (95% credible interval) in kilometers between Ref. [61] and this work for different EOS constraints. The bottom row refers to the posterior using all listed constraints jointly.

Constraint	$R_{1.4}$ [km]	
	Ref. [61]	This work
$\chi_{\text{EFT}}$	$12.11^{+1.69}_{-3.39}$	$12.59^{+2.24}_{-3.51}$
Radio timing	$13.70^{+1.41}_{-2.17}$	$13.71^{+1.19}_{-1.88}$
PSR J0030+0451	$13.17^{+1.65}_{-2.24}$	$13.48^{+1.42}_{-2.15}$
PSR J0740+6620	$13.39^{+1.57}_{-1.72}$	$13.79^{+1.26}_{-1.73}$
GW170817	$11.90^{+1.78}_{-1.74}$	$12.24^{+2.03}_{-2.18}$
All	$12.26^{+0.80}_{-0.91}$	$12.62^{+0.97}_{-1.11}$

maximal central density inside a NS,  $n_{\text{TOV}}$ , as shown by Table III in Appendix A.

After validating that our framework agrees with previously published results, we now assess the runtime and its scaling as a function of the dimensionality of the EOS parameter space. Using a single NVIDIA H100 GPU, the inferences listed in Table II were performed under 1 hour of wall time. Solving the TOV equations to obtain the macroscopic properties of NSs (100 masses and their corresponding radii, and tidal deformabilities) for a single proposal takes around  $\sim 0.3$  milliseconds, which is on par with previous works relying on machine learning surrogates (see, e.g., Refs [80–82] for recent examples using neural networks), however, without the need of pretraining a machine learning model in our case.

Future experiments are expected to provide more precise observations of NS, which could require a more flexible EOS parameterization to avoid biases. This motivates us to extend the parameter space by adding more CSE grid points and assess the impact on the scaling of our sampling efficiency. Specifically, we perform Bayesian inference jointly on all EOS constraints considered in this work by testing CSE grids with 10, 20, and 30 points, respectively. Figure 2 shows the runtime per effective sample size (ESS), estimated with Geyer’s initial monotone sequence criterion [195, 196]. The axis on the right converts this number to an estimate of the total wall time required in case one requires 5000 effective samples. We perform the scaling runs on two different GPU architectures, namely an NVIDIA A100 and NVIDIA H100, and show the average and standard deviation over all the EOS parameters. The parametrization with the highest number of grid points increases the runtime roughly by a factor 4 to produce a similar number of effective samples. Therefore, our inferences still only require a couple of hours to provide accurate posteriors on the EOS, even when using a more flexible parametrization.

<sup>7</sup>The quoted results for Ref. [61] are obtained from [https://multi-messenger.physik.uni-potsdam.de/eos\\_constraints/](https://multi-messenger.physik.uni-potsdam.de/eos_constraints/).

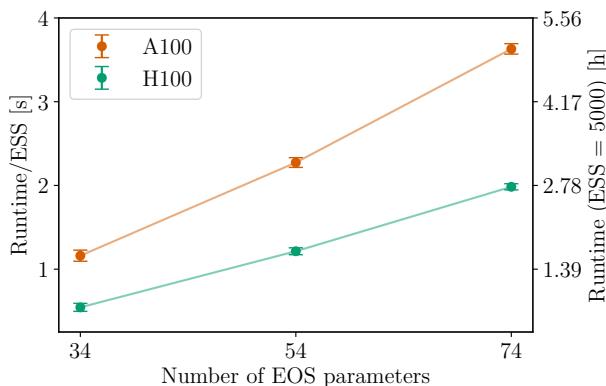


FIG. 2: Scaling of the runtime divided by effective sample size (ESS) as a function of the number of EOS parameters when varying the number of grid points for the CSE parameterization, for two different GPU architectures. The right axis converts this number to total wall time to obtain 5000 effective samples.

### B. Constraining $n_{\text{break}}$ with neutron star data

The EOS parametrization employed in this work takes the breakdown density  $n_{\text{break}}$  of the metamodel as a parameter that can be varied freely. This density is an indicator for the densities for which a simple nucleonic description of the EOS is not sufficient anymore. We can use our framework to constrain  $n_{\text{break}}$  using NS data and directly probe this breakdown with astrophysical information. To this end, we perform Bayesian inference as explained above with two key modifications. First, we use a wider, agnostic prior on  $n_{\text{break}}$ , namely a uniform distribution between  $[1n_{\text{sat}}, 4n_{\text{sat}}]$ . Second, we only consider EOS constraints involving NS data, i.e., we do not use the constraint from  $\chi_{\text{EFT}}$ . Figure 3 shows the marginal posterior distribution of  $n_{\text{break}}$ . We infer a breakdown density of  $n_{\text{break}} = 2.47^{+1.53}_{-1.02} n_{\text{sat}}$  at the 90% credible level. Hence, NS data prefers a smooth nucleonic EOS up to 2-3 times the nuclear saturation density, a range within the reach of microscopic nuclear theory [51]. While  $n_{\text{break}}$  is not tightly constrained by the data considered in this work, future experiments providing more NS data can potentially improve its precision. Nevertheless, our result serves as proof of concept that the metamodel breakdown density can be determined on-the-fly with Bayesian inference.

We point out that Ref. [197] similarly inferred this transition density. However, comparing the two results is complicated, as they use different parameterizations and prior choices. In particular, we sample the  $n_{\text{break}}$  parameter on-the-fly during inference, whereas Ref. [197] infers this parameter by considering Bayes factors of different runs with fixed  $n_{\text{break}}$ . Moreover, we extend the MM parametrization to higher orders and consider a speed-of-sound parametrization at high densities, rather than piecewise polytropes.

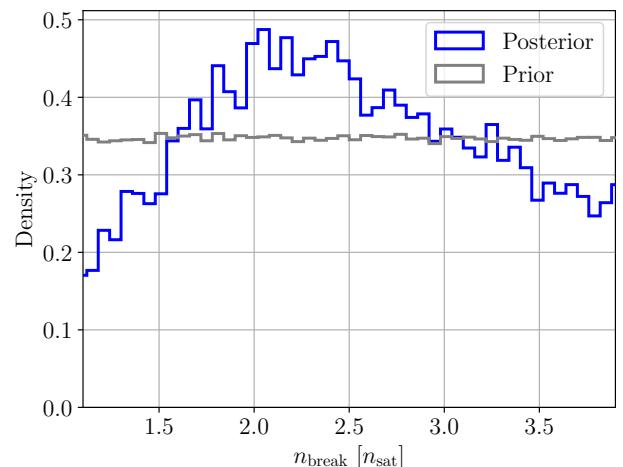


FIG. 3: Posterior on  $n_{\text{break}}$  inferred from a uniform prior between  $1n_{\text{sat}}$  to  $4n_{\text{sat}}$  (other priors are given in Table I) and given the NS observations from Table II.

### C. Constraints on nuclear empirical parameters

Currently available NS observations do not yet tightly constrain the EOS, as demonstrated by the previous section. In this section, we instead consider a hypothetical scenario where observations of NSs are pushed to extreme limits and uncertainties on the measured radii and tidal deformabilities can be ignored. We aim to obtain a qualitative estimate of how highly accurate knowledge of the properties of NSs translates into constraints on the NEPs, since such constraints can then be compared to those obtained from nuclear experiments. For simplicity, we restrict our parametrization to the metamodel part and no longer use the CSE at high densities. We enforce all these metamodel EOSs to be causal and therefore truncate the EOS once the speed-of-sound reaches 1. Otherwise, the EOSs are truncated at the maximal central density allowed in an NS.

We generate a mock “true” EOS with a TOV mass of roughly  $2.0M_{\odot}$  and aim to recover its EOS parameters from its  $M$ - $\Lambda$  curve using the gradient-based optimization scheme discussed in Sec. II F, leaving a full Bayesian inference-based study for future work. We perform multiple such recoveries where we change the number of NEPs that are varied freely in the recovery, starting with only the first order parameter  $L_{\text{sym}}$ . In subsequent recovery runs, we also allow the optimizer to explore the higher-order parameters, each time adding the parameters of the next order of the Taylor expansions of Eqs. (2)-(3). For simplicity, we do not vary the  $E_{\text{sym}}$  parameter, since it has a small impact on the EOS at high densities, and we wish to focus on the parameters entering at higher orders of the Taylor expansion. For each recovery, we start the optimizer from 100 random EOS samples, sampled from the priors shown in Table I, and perform at most 1000

iterations with a fixed learning rate of 1.<sup>8</sup> For the final result, we keep only those EOS samples that within the  $[1 M_{\odot}, 2 M_{\odot}]$  mass range deviate less than 100 meters in radius from the “true”  $M$ - $R$  curve and less than 10 in tidal deformability from the “true”  $M$ - $\Lambda$  curve. Figure 4 shows the resulting recovery on the slope of the symmetry energy  $L_{\text{sym}}$ . The range of the recovered  $L_{\text{sym}}$  values significantly depends on the number of NEPs that are allowed to vary. When  $L_{\text{sym}}$  is the only degree of freedom, we recover the value of the “true” EOS, with a small width of around 0.5 MeV, as expected. However, when more NEPs are varied, this range increases. Nevertheless, the middle and right panels of Fig. 4 demonstrate (for the case where all NEPs are free parameters) that the corresponding EOSs and NS configurations agree well with the true EOS. Therefore, from our gradient-based optimization approach, we conclude that, while accurate observations of NSs allow us to tightly constrain the EOS itself, the corresponding constraints on the NEPs suffer from a degeneracy in our metamodel parametrization, which was already hinted at by Refs. [73, 198–201]. Therefore, fitting relations between  $L_{\text{sym}}$  and  $R_{1.4}$  previously proposed in the literature, e.g., Ref. [202], will need to be assessed carefully in light of the assumptions regarding the parametrization used to obtain them. Finally, we note that this degeneracy is expected to be more pronounced if we include the CSE parametrization at higher densities.

## IV. DISCUSSION

### A. Equation of state degeneracies

Our results from Sec. III C show that the inverse problem of NSs is sensitive to a degeneracy in our metamodel parametrization. While this was shown using our gradient-based optimization scheme, future work will assess if these degeneracies remain when using traditional stochastic sampling algorithms. A similar degeneracy was already observed in previous works that used stochastic samplers, e.g., Refs. [73, 198–201]. However, our work extends these studies by simultaneously considering (i) higher-order NEPs, (ii) a wide range of NS masses, (iii) high accuracy in the recovered EOSs, highlighting the problem’s relevance for future experiments, and (iv) the dependence as a function of the parametrization. While our analysis in Sec. III C served as a theoretical exploration of this degeneracy, we expect that a full Bayesian analysis employing future constraints from nuclear theory and experiments on the NEPs, especially for the higher-order parameters of the MM, alleviates this issue.

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<sup>8</sup>The runs are performed on an AMD EPYC 7543 VM CPU and take at most a few minutes per recovery.

We expect the level of degeneracy to be different for different EOS models, and we will explore such model dependencies in future work. Furthermore, we will also investigate the potential introduction of similar systematic effects originating from certain aspects neglected in this study, such as the crust treatment (see, e.g., Refs. [203–208]), and consider unified EOS models [209] as alternative.

### B. Next-generation detectors

Future detectors will generate an unprecedented volume of NS observations. Each event provides valuable constraints on the EOS, yet, when analyzed with existing methods, at substantial economic and environmental costs [210]. Our work addresses this pressing data analysis challenge by employing differentiable programming and GPU accelerators. Using JAX, analyzing a GW signal from a BNS merger to obtain masses and tidal deformabilities can be completed within minutes [185], after which we obtain posteriors on the EOS in less than 1 hour on an NVIDIA H100 GPU. Crucially, we do not require pre-trained machine-learning emulators of the TOV equations, thereby ensuring accuracy in the results and further reducing computational costs. Therefore, our methods are a key advancement towards handling the data analysis problems offered by future detectors.

Furthermore, while our constraint on the  $n_{\text{break}}$  parameter from Sec. III B serves as a proof of principle, we expect that additional observations of NSs will tighten this constraint and allow us to test the consistency between nuclear physics and astrophysics. Future research using simulated data will have to investigate to what accuracy the breakdown density of the metamodel can in fact be constrained from NS data. Moreover, the proof of concept presented here can be extended to directly test predictions from  $\chi$ EFT, providing an analysis complementary to Ref. [51] with an alternative EOS parametrization to assess possible systematic effects in such an analysis.

### C. Extension to other analyses

In this work, we only focused on inferring the EOS from NS observations. However, other inferences require to jointly model the EOS together with additional effects or parameters. Naturally, these analyses can therefore benefit from the acceleration offered by the methods presented in this work.

For instance, we can infer the EOS simultaneously with cosmological parameters or population models (see, e.g., Refs. [82, 211, 212]). Moreover, our method can be extended to jointly infer the parameters of modified gravity, either for a specific theory (see Ref. [213] for an overview) or for a parametrized post-TOV formalism [214–216]. Our gradient-based scheme can be used to explore this extended parameter space and uncover degeneracies be-

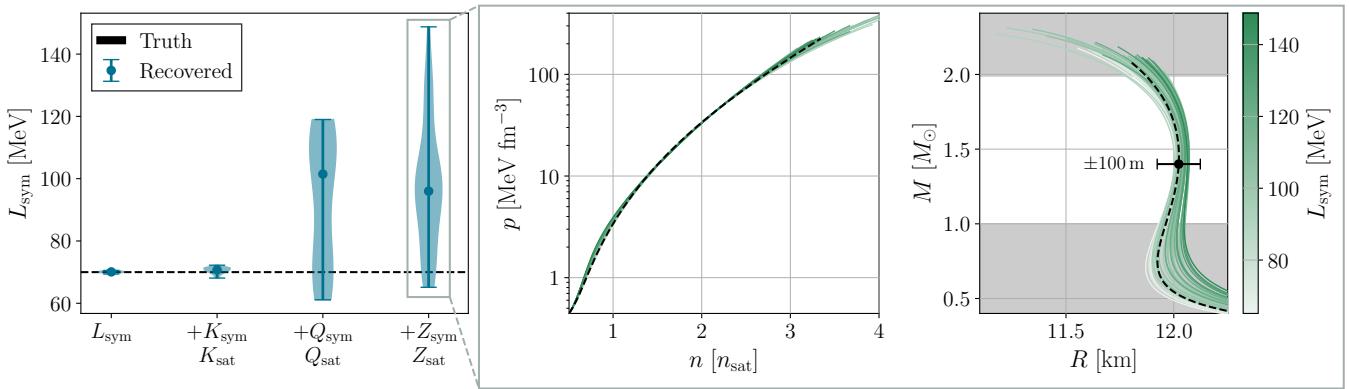


FIG. 4: Recovery of an EOS in the metamodel parametrization from the  $M$ - $\Lambda$  curve using the gradient-based optimization algorithm as a function of the number of NEPs being varied. The left panel shows the recovered range of  $L_{\text{sym}}$  when increasingly more NEPs at higher orders in the Taylor expansion are varied freely. For the recovery in which all NEPs were varied freely, we show the recovered EOS samples in the middle and right panels. The middle panel shows the pressure as a function of density, while the right panel shows the mass-radius profile of NSs, color-coded by their  $L_{\text{sym}}$  value. The grey bands show mass ranges not considered by the loss function in Eq. (13). All EOSs considered show a deviation with respect to the target of  $< 100$  meters in radius and  $< 10$  in dimensionless tidal deformabilities across the mass range considered in the loss function of Eq. (13).

tween the EOS and deviations from general relativity in observations of NSs [217]. Similar degeneracies can arise from other effects, such as phase transitions [218–220] or dark matter [221], which can also be constrained with either Bayesian inference or the gradient-based method.

Going beyond extreme matter studies, other analyses in related fields that have to solve ordinary differential equations on the fly during inference can benefit from the methods discussed in this work, such as the evaluation of effective one-body gravitational waveforms [222], solving the lens equation in strong lensing analyses (see, e.g., Refs. [223, 224]), and nuclear reaction networks [225].

## V. CONCLUSION

The inverse problem of neutron stars (NSs), i.e., inferring the nuclear equation of state (EOS) from NS data, is a computationally challenging task, making it difficult to understand the interplay between nuclear physics and astrophysics. Moreover, the prospect of a large volume of NS observations delivered by future detectors calls for an efficient and scalable way of addressing this inverse problem. In this work, we have presented a solution based on differentiable programming, leveraging the automatic differentiation capabilities and support for hardware accelerators, such as graphical processing units (GPUs), from JAX. By using gradient-based samplers, we can perform Markov chain Monte Carlo (MCMC) sampling on high-dimensional EOS parameter spaces in under an hour of wall time on a single NVIDIA H100 GPU without resorting to machine learning emulators for the Tolman-Oppenheimer-Volkoff (TOV) equations. Moreover, we provide a proof of concept on how the breakdown of the

metamodel description can be inferred from NS data on the fly during sampling.

Additionally, our framework introduces a novel optimization scheme that recovers the EOS of a given mass-radius or mass-tidal deformability relation by performing gradient descent on a loss function formulated in terms of the EOS parameters, differentiating through the TOV equations. Using this tool, we have discussed a degeneracy present in our metamodel parametrization, which might affect inferences of nuclear empirical parameters from detailed knowledge of NSs using certain EOS parametrizations.

In short, our methods leveraging differentiable programming address the data analysis challenges of future detectors and offer a pragmatic way to investigate the relation between EOS parameterizations and properties of NSs.

## Appendix A: Method validations

In Table III, we extend the results of Table II and show  $M_{\text{TOV}}$ , the pressure at  $3n_{\text{sat}}$ ,  $p(3n_{\text{sat}})$ , and the maximal central density  $n_{\text{TOV}}$  at the core of an NS, between Ref. [61] and this work for the EOS constraints discussed in Sec. II E.

## DATA AVAILABILITY

Our EOS code and TOV solver are available open source at <https://github.com/nuclear-multimessenger-astronomy/jester>. The code used in this study is available at [https://github.com/ThibreauWouters/paper\\_jester](https://github.com/ThibreauWouters/paper_jester).

TABLE III: Comparison of inferred  $M_{\text{TOV}}$ , the pressure at  $3n_{\text{sat}}$ ,  $p(3n_{\text{sat}})$ , and the maximal central density  $n_{\text{TOV}}$  at the core of an NS, between Ref. [61] and this work for various EOS constraints.

Constraint	$M_{\text{TOV}} [M_{\odot}]$		$p(3n_{\text{sat}}) [\text{MeV fm}^{-3}]$		$n_{\text{TOV}} [n_{\text{sat}}]$	
	Ref. [61]	This work	Ref. [61]	This work	Ref. [61]	This work
$\chi_{\text{EFT}}$	$2.05^{+1.08}_{-1.16}$	$2.03^{+1.03}_{-0.97}$	$69^{+186}_{-53}$	$72^{+165}_{-65}$	$6.51^{+10.7}_{-3.11}$	$6.53^{+9.26}_{-4.09}$
Radio timing	$2.35^{+0.73}_{-0.29}$	$2.20^{+0.39}_{-0.26}$	$111^{+140}_{-49}$	$97^{+65}_{-49}$	$5.51^{+1.89}_{-1.66}$	$5.68^{+1.63}_{-1.74}$
PSR J0030+0451	$2.16^{+0.83}_{-0.71}$	$2.19^{+0.78}_{-0.76}$	$89^{+143}_{-46}$	$94^{+135}_{-62}$	$5.62^{+4.44}_{-1.91}$	$5.60^{+3.65}_{-2.59}$
PSR J0740+6620	$2.34^{+0.65}_{-0.32}$	$2.38^{+0.70}_{-0.42}$	$107^{+125}_{-40}$	$118^{+150}_{-59}$	$5.34^{+1.63}_{-1.61}$	$5.16^{+1.72}_{-2.04}$
GW170817	$1.90^{+0.71}_{-0.41}$	$1.93^{+0.73}_{-0.48}$	$59^{+91}_{-25}$	$62^{+109}_{-42}$	$7.37^{+3.64}_{-2.72}$	$7.24^{+3.77}_{-3.21}$
All	$2.25^{+0.42}_{-0.22}$	$2.23^{+0.41}_{-0.23}$	$90^{+71}_{-31}$	$92^{+70}_{-42}$	$5.92^{+1.35}_{-1.38}$	$5.92^{+1.38}_{-1.49}$

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