Nonreciprocal wave-mediated interactions power a classical time crystal

Mia C. Morrell*, Leela Elliott*, and David G. Grier
Department of Physics and Center for Soft Matter Research,
New York University, New York, NY, 10003, USA
(Dated: April 23, 2025)

An acoustic standing wave acts as a lattice of evenly spaced potential energy wells for sub-wavelength-scale objects. Trapped particles interact with each other by exchanging waves that they scatter from the standing wave. Unless the particles have identical scattering properties, their wave-mediated interactions are nonreciprocal. Pairs of particles can use this nonreciprocity to harvest energy from the wave to sustain steady-state oscillations despite viscous drag and the absence of periodic driving. We show in theory and experiment that a minimal system composed of two acoustically levitated particles can access five distinct dynamical states, two of which are emergently active steady states. Under some circumstances, these emergently active steady states break spatiotemporal symmetry and therefore constitute a classical time crystal.

Waves exert forces on scatterers. Scattered waves coalesce collections of independent scatterers into self-organizing dynamical systems by mediating interparticle interactions [1–7]. Wave-mediated pair interactions are not constrained by Newton's third law because the scatterers are not a closed system: scattered waves can carry away momentum, causing pairs of interacting particles to recoil [8–11]. The nonreciprocity of wave-mediated interactions allows scatterers to capture energy from the wave and to transduce it into collective motion, thereby endowing an otherwise passive system with the defining characteristics of active matter [12–15]. This form of activity is distinctive because it is not an inherent property of the individual particles, but instead is an emergent property of the particles' state of organization [11].

Here we demonstrate that emergent activity can take the form of sustained oscillations in an acoustically levitated array of particles. Nonreciprocal wave-mediated interactions transfer energy from the levitator's standing wave into the lattice's normal modes without periodic driving. We show both in theory and through experiments that a minimal array of two particles can use this mechanism to access a variety of dynamical states, including an active steady state [16–18], that spontaneously breaks spatiotemporal symmetry and therefore constitutes a classical time crystal [10, 19, 20]. Figure 1(a) shows such a time crystal in action.

The levitator used for this study is based on the popular TinyLev2 design [21], which operates at $40\,\text{kHz}$ in air and creates a linear array of pressure nodes with a lattice constant of $4.3\,\text{mm}$. Each node acts as a three-dimensional potential energy well for a millimeter-scale bead [22, 23] and exerts a nearly Hookean restoring force whose stiffness depends on the bead's properties. Trapped beads interact by exchanging scattered waves, as depicted schematically in Fig. 1(b).

The standing pressure wave along the levitator's axis,

$$p(z,t) = p_0 \sin(kz) \cos(\omega t), \tag{1}$$

is characterized by its amplitude, p_0, its frequency, ω,

and its wave number, $k = \omega/c_0$, in a medium of sound speed c_0. Objects scattering this wave experience time-averaged forces that organize them into a periodic lattice along the wave's axis, $\hat{z}$, even if they have different wave-scattering characteristics. Heterogeneity in the trapped particles' properties constitutes quenched disorder in the levitated array and creates a context for nonreciprocal dynamics [11] that have not been addressed previously.

A sphere labeled j and located at z_j within the standing wave experiences a time-averaged force originally formulated by Gor'kov [24–26],

$$\boldsymbol{F}_j(kz_j) = -\frac{\pi}{3} F_0\, A_j x_j^3 \, \sin(2kz_j)\, \hat{z}, \tag{2a}$$

whose scale,

$$F_0 = \frac{p_0^2}{\rho_0 \omega^2}, \tag{2b}$$

is proportional to the acoustic energy density in a medium of mass density ρ_0. Our levitator's calibrated [23] force scale is $F_0 = (24.7 \pm 0.5)\,\text{μN}$. The Gor'kov force on a sphere also depends on the sphere's radius, a_j, relative to the wavelength of sound,

$$x_j = ka_j, \tag{2c}$$

as well its density, ρ_j, and the ratio of its isentropic compressibility, κ_j, to that of the medium, κ_0. These material properties contribute to a dimensionless coupling constant [11],

$$A_j = f_0^{(j)} + f_1^{(j)}, \tag{2d}$$

that combines the spheres' monopole (pressure) and dipole (velocity) polarizabilities,

$$f_0^{(j)} = 1 - \frac{\kappa_j}{\kappa_0} \quad \text{and} \tag{3a}$$

$$f_1^{(j)} = \frac{\rho_j - \rho_0}{2\rho_j + \rho_0}, \tag{3b}$$

FIG. 1. (a) Experimental realization of a steady-state time crystal composed of two millimeter-scale spheres of expanded polystyrene levitated in air by an acoustic standing wave at 40 kHz. Images captured at 170 frames/s reveal sustained oscillations without periodic driving and despite dissipation due to viscous drag. Similar oscillations also arise spontaneously in larger arrays of spheres with differing diameters. Curly braces identify beats between the particles' symmetric mode and the traps' natural frequencies. (b) Model for the forces acting on spheres localized at the nodes of an acoustic standing wave, including restoring forces, $\boldsymbol{F}_j$, and interparticle forces, $\boldsymbol{F}_{ij}$, mediated by exchange of scattered waves.

respectively. Rigid spheres ($\kappa_j \ll \kappa_0$) that are denser than the medium ($\rho_j > \rho_0$) tend to be stably trapped at nodes of the pressure field, located at $kz_j = j\pi$. This is the case for the expanded polystyrene (EPS) beads depicted in Fig. 1(a) that are sufficiently incompressible relative to air that we set $f_0^{(j)} \approx 1$. The beads' mass density is $\rho_j = (30.5 \pm 0.2)\,\text{kg/m}^3$ [23, 27], from which we obtain $f_1^{(j)} = 0.456 \pm 0.001$.

The wave-mediated interaction force was originally formulated by König in 1893 [28] for the special case of identical spheres in a common nodal plane. This result recently has been generalized to accommodate dissimilar spheres at arbitrary positions within the wave [11]. The leading-order generalized König force on sphere j due to the wave scattered by sphere i,

$$\boldsymbol{F}_{ji}^K(k\boldsymbol{r}_{ji}) = -2\pi F_0\, f_1^{(i)} f_1^{(j)} x_j^3 x_i^3\, \Phi(k r_{ji})\, \hat{r}_{ji}, \quad (4a)$$

depends on the spheres' separation, $\boldsymbol{r}_{ji} = \boldsymbol{r}_j - \boldsymbol{r}_i$, through the dimensionless geometric factor, $\Phi(k\boldsymbol{r})$. For spheres trapped along the axis of a standing wave,

$$\Phi(k\boldsymbol{r}) = \frac{\cos(kz) + kz\,\sin(kz)}{(kz)^2}. \quad (4b)$$

Whereas the transverse König interaction falls off as r^{-4} [11, 22, 28, 29], the axial König force falls off as r^{-2} and thus is longer-ranged. The expression in Eq. (4) for the axial pair interaction does not appear to have been reported previously, although its qualitative features have been noted [30]. Equation (4) is strictly valid only in the Rayleigh regime, which pertains to scatterers that are smaller than the wavelength of sound, $x_j < 1$. Comparison with numerical studies [31], however, shows that Eq. (4) captures qualitative features of the inter-particle force for larger spheres in the range $1 \le x_j < 3$.

While Equation (4) is reciprocal under exchange of the indices i and j [30, 32], incorporating quadrupolar and octupolar scattering at leading order in x_j [11] yields corrections, χ_{ji}, to the pair interaction,

$$\boldsymbol{F}_{ji}(kz) = \boldsymbol{F}_{ji}^K(kz)\,(1 - \chi_{ji}), \quad (5a)$$

that are nonreciprocal unless spheres i and j are identical. For spheres made of the same material, these higher-order corrections reduce to two contributions:

$$\chi_{ji} = \frac{2}{5}\,\sigma_{ji}^{(2)} + \frac{1}{10}\Delta_{ji}^{(2)}, \quad (5b)$$

that depend on the spheres' sizes through

$$\sigma_{ji}^{(n)} = x_j^n + x_i^n \quad \text{and} \quad (5c)$$

$$\Delta_{ji}^{(n)} = x_j^n - x_i^n. \quad (5d)$$

The latter of these factors changes sign under exchange of indices and therefore describes a nonreciprocal contribution to the pair interaction. More generally, Eq. (5d) establishes that spheres of different sizes interact nonreciprocally, as depicted schematically in Fig. 1(b).

Combining the Gor'kov force from Eq. (2) with the generalized axial König interaction from Eq. (5) yields the equations of motion for an array of acoustically levitated spheres trapped at the nodes of a standing wave. The dimensionless displacement of the j-th sphere from its trap, $\zeta_j(\tau) = kz_j(t) - j\pi$, evolves in time according to the system of coupled equations,

$$\nu_j \equiv \dot{\zeta}_j \quad (6a)$$

$$\dot{\nu}_j = -\frac{1}{2}\sin(2\zeta_j) + \sum_{i \ne j} B_{ji}\,\Phi(kz_{ji}) - \Gamma_j\,\nu_j. \quad (6b)$$

Dots in Eq. (6) represent derivatives with respect to the dimensionless time, $\tau = \Omega_0 t$, which is scaled by the natural oscillation frequency,

$$\Omega_0 = \sqrt{\frac{F_0 A_j k^4}{2\rho_j}}. \quad (7)$$

For the EPS beads in our system, $\Omega_0 = (415 \pm 2)\,\text{rad/s}$ or $(66.1 \pm 0.3)\,\text{Hz}$. The interparticle coupling,

$$B_{ji} = 3\frac{f_1^{(j)2}}{A_j}\,x_i^3\,(1 - \chi_{ji})\,\hat{r}_{ji} \cdot \hat{z}, \quad (8)$$

FIG. 2. Power spectral density [33] of the symmetric (blue) and antisymmetric (red) modes of two EPS beads actively oscillating in an acoustic levitator. (a) Small particles ($ka_1 = 0.85$ and $ka_2 = 1.08$) oscillate in the symmetric mode at the predicted common-mode frequency, Ω_0. (b) Larger particles ($ka_1 = 1.2$ and $ka_2 = 1.3$) break spatiotemporal symmetry with antisymmetric oscillations at a lower frequency. Dashed lines are predictions of Eq. (11) for the normal-mode frequencies. The power spectrum of a single-particle trajectory (gray) represents the measurement's noise threshold.

is nonreciprocal if the spheres differ in size.

For simplicity, we model dissipation by Stokes drag acting on the individual spheres indepependently in a medium of viscosity η_0. Its influence is characterized by a dimensionless damping rate, $\Gamma_j = 2\epsilon\, x_j^{-2}$, that is scaled by the natural frequency through

$$\epsilon = \frac{9}{4} \frac{k^2 \eta_0}{\rho_j \,\Omega_0}. \tag{9}$$

Taking the viscosity of air to be $\eta_0 = 1.825 \times 10^{-5}\,\mathrm{Pa\,s}$ [34] yields $\epsilon = (1.74 \pm 0.01) \times 10^{-3}$ for our system. The expression for Γ_j can be modified to account for inertial effects in the fluid medium [23, 29, 35, 36] without affecting the functional form of Eq. (6).

The normal modes of a pair of acoustically levitated spheres, labeled 1 and 2, are obtained from the Jacobian of the equations of motion, $\underline{J}(\zeta_1, \zeta_2; \nu_1, \nu_2)$, by solving the characteristic equation,

$$|\underline{J}(0,0;\nu_1,\nu_2) - \lambda\underline{I}| = 0, \tag{10}$$

for the eigenvalues, λ, at the fixed point for the Gor'kov force, $\zeta_1 = \zeta_2 = 0$. The full analytic solutions for λ are unwieldy, even in the two-particle case. For clarity, we treat drag as a perturbation in the limit of small viscosity ($\epsilon < 1$) [37] and obtain two pairs of eigenvalues, $\lambda_\pm^{(s)}$ and

$\lambda_\pm^{(a)}$, the former being associated with symmetric modes,

$$\frac{\lambda_\pm^{(s)}}{\Omega_0} = \pm i - \epsilon\, \frac{\Lambda(1)}{\Lambda(3)} + \mathcal{O}\left\{\epsilon^2\right\}, \tag{11a}$$

and the latter with antisymmetric modes,

$$\frac{\lambda_\pm^{(a)}}{\Omega_0} = \pm i\sqrt{1 + g\,\Lambda(3)} - \frac{\epsilon}{x_1^2 x_2^2}\frac{\Lambda(5)}{\Lambda(3)} + \mathcal{O}\left\{\epsilon^2\right\}. \tag{11b}$$

The particles' sizes determine the nature of these solutions through stability functions,

$$\Lambda(n) = \left(1 - \frac{2}{5}\sigma_{12}^{(2)}\right)\sigma_{12}^{(n)} + \frac{1}{10}\Delta_{12}^{(2)}\Delta_{12}^{(n)}, \tag{11c}$$

that establish bounds between the system's different dynamical states. The frequency of the antisymmetric mode depends on particle size through a factor,

$$g = 3\frac{\pi^2 - 2}{\pi^3}\frac{f_1^{(j)\,2}}{A_j}, \tag{11d}$$

that depends on the spheres' composition. The normal mode frequencies, $f_\pm^{(s,a)} = \frac{1}{2\pi}\mathrm{Im}\left\{\lambda_\pm^{(s,a)}\right\}$, are not influenced by drag to leading order in ϵ.

Figure 2 presents the power spectral density, $S(f)$, of the common and relative motions of pairs of particles suspended in the acoustic levitator [33] for comparatively small spheres (Fig. 2(a)) that are predicted to be in the symmetric oscillator state and for larger spheres (Fig. 2(b)) whose stronger nonreciprocal interactions are predicted to select a time crystal. Equation (11a) suggests that the frequency of the symmetric common mode, $f_\pm^{(s)} = \Omega_0$, should be independent of size. This is consistent with the strong spectral peaks at Ω_0 for both systems. The frequency of the antisymmetric mode is predicted to be higher than the natural frequency, $f_\pm^{(a)} > \Omega_0$, for the smaller spheres in Fig. 2(a), and no spectral feature is observed at that frequency. The larger spheres are predicted to have a lower antisymmetric-mode frequency, $f_\pm^{(a)} < \Omega_0$, and indeed a small spectral peak is observed at the predicted frequency in Fig. 2(b). This is consistent with expectations for a classical time crystal.

The five distinct dynamical states of the weakly-damped two-particle system are delineated by the conditions $\Lambda(1) = 0$, $\Lambda(3) = 0$, $\Lambda(5) = 0$ and $\Lambda(3) = -g^{-1}$, which are plotted on the state diagram in Fig. 3(a). These are superimposed on the full analytic solutions for the eigenvalues, $\lambda_+^{(s,a)}$, as a function of particle size. The stability functions are positive and small ($0 < \Lambda(n) < 1$) in the Rayleigh regime ($x_j < 1$) which means that small particles undergo damped harmonic oscillations in superpositions of normal-mode frequencies.

Larger particles behave qualitatively differently, especially when they differ in size. The first new dynamical

FIG. 3. (a) Dynamical states predicted by Eq. (11) for two levitated spheres as a function of a_1 and a_2. Symmetric oscillator: $\Lambda(3) > 0$ (blue); Antisymmetric time crystal: $\Lambda(3) < 0$ (red); Unstable: $\Lambda(3) < g^{-1}$ (gray). Active oscillations arise in the (yellow) region bounded by $\Lambda(1) = 0$ and $\Lambda(5) = 0$. White circles denote conditions from Fig. 1(a), Fig. 2(a) and Fig. 2(b). Rayleigh limit: black dashed curve. (b) Frequency and (c) growth rate from Eq. (10) for $a_1 = 1$ mm. Curves represent symmetric (blue) and antisymmetric (red) modes. Shading indicates dynamical state: oscillator (blue), time crystal (red), active (yellow) and unstable (gray).

state appears when $\Lambda(1)$ changes sign. Activity compensates for drag in the symmetric common mode when $\Lambda(1) = 0$, which marks a transition into an active oscillatory state with a positive growth rate, $\mathrm{Re}\left\{\lambda_{\pm}^{(s)}\right\} \geq 0$. Higher-order contributions to the damping can stabilize this mode, which therefore settles into steady-state os-

cillation at the natural frequency, Ω_0 [23]. The antisymmetric mode is damped under these conditions.

The active oscillator transitions abruptly into an active time crystal when $\Lambda(3)$ changes sign. At this point, the antisymmetric mode flips into the active state and the symmetric common mode becomes maximally damped. The frequency of the antisymmetric mode simultaneously dips below the frequency of the symmetric mode ($\Omega < \Omega_0$), which signals breaking of both spatial and temporal symmetry and is the defining characteristic of a time crystal [38, 39]. Steady-state oscillations of the acoustically levitated time crystal are actively sustained by nonreciprocal interactions and are stabilized by drag.

Activity emerges in the limited range of particle sizes and size ratios bounded by $\Lambda(1) = 0$ and $\Lambda(5) = 0$. These conditions converge when $x_1 = x_2 = \sqrt{5/4}$, because identical spheres lack the nonreciprocal interactions that power emergently active states. Damping surpasses activity for spheres that are large enough that $\Lambda(5) < 0$, although the system retains its time crystal nature because the antisymmetric mode is lower-frequency and longer-lived than the symmetric mode.

Further increasing particle sizes to the point that $\Lambda(3) \leq -g^{-1}$ destabilizes the system altogether. The König interaction is repulsive and exceeds the maximum trapping force for such large spheres. The initial instability involves the system passing through a bifurcation, which is reminiscent of the emergence of chiral states in other systems displaying time crystals [40].

Our model for the two-particle system relies on a form for the pair interaction from Eq. (5) that is appropriate for small particles. Numerical evaluation of the eigenvalues of the full steady-state Jacobian confirm this model's predictions for $x_j < 3$. The data in Fig. 3(b) and (c) are computed with the full model for a pair of spheres whose smaller partner has a radius of $a_1 = 1$ mm ($x_1 = 0.72$) and show that a steady-state time crystal occurs at size ratio $a_2/a_1 = 2.1$, which is consistent with the leading-order small-drag approximation.

As anticipated by Ref. [39], our system does not rely on thermal activation to break spatiotemporal symmetry or to maintain a steady state. Its actively sustained antisymmetric oscillations therefore embody a true classical time crystal [38, 39, 41] that is stabilized by damping [41–43]. Classical time crystals have been demonstrated in theoretical studies of hydrodynamic flocking [44], in simulations of active colloids [45] and nonreciprocal Ising models [46], and in experimental studies of interacting robots [44]. In all of these studies, the nonreciprocity responsible for time-crystal dynamics is programmed rather than emergent. Emergently active classical time crystals have recently been observed in photonic metamaterials in experiment [20] and simulation [10], but this study is the first to quantitatively and completely explain how classical time crystals arise and achieve activity with both analytical theory and experiment.

While we have focused on a pair of acoustically levitated particles as a minimal model system, emergent activity also arises in larger lattices of acoustically trapped particles. In long chains, quenched disorder not only creates the conditions required for sustained oscillations, but also mediates transitions between propagation and localization. We will address these themes in future studies.

---

[1] M. M. Burns, J.-M. Fournier, and J. A. Golovchenko, Optical binding, Phys. Rev. Lett. **63**, 1233 (1989).

[2] M. M. Burns, J.-M. Fournier, and J. A. Golovchenko, Optical matter: crystallization and binding in intense optical fields, Science **249**, 749 (1990).

[3] P. Marston, E. Trinh, J. Depew, and J. Asaki, Response of bubbles to ultrasonic radiation pressure: Dynamics in low gravity and shape oscillations, in *Bubble Dynamics and Interface Phenomena*, Fluid Mechanics and its Applications, Vol. 23, edited by J. Blake, J. Boulton-Stone, and N. Thomas (Springer Science+Budiness Media, 1994) pp. 343–353.

[4] D. Rabaud, P. Thibault, M. Mathieu, and P. Marmottant, Acoustically bound microfluidic bubble crystals, Phys. Rev. Lett. **106**, 134501 (2011).

[5] A. Doinikov and S. Zavtrak, On the "bubble grapes" induced by a sound field, J. Acoust. Soc. Am. **99**, 3849 (1996).

[6] M. X. Lim, B. VanSaders, A. Souslov, and H. M. Jaeger, Mechanical properties of acoustically levitated granular rafts, Phys. Rev. X **12**, 021017 (2022).

[7] M. X. Lim, B. VanSaders, and H. M. Jaeger, Acoustic manipulation of multi-body structures and dynamics, Rep. Prog. Phys. (2024).

[8] S. Sukhov, A. Shalin, D. Haefner, and A. Dogariu, Actio et reactio in optical binding, Opt. Express **23**, 247 (2015).

[9] S. Sukhov and A. Dogariu, Non-conservative optical forces, Rep. Prog. Phys. **80**, 112001 (2017).

[10] V. Raskatla, T. Liu, J. Li, K. F. MacDonald, and N. I. Zheludev, Continuous space-time crystal state driven by nonreciprocal optical forces, Phys. Rev. Lett. **133**, 136202 (2024).

[11] E. M. King, M. C. Morrell, J. B. Sustiel, M. Gronert, H. Pastor, and D. G. Grier, Scattered waves fuel emergent activity, Phys. Rev. Research **7**, 013055 (2025).

[12] S. Ramaswamy, The mechanics and statistics of active matter, Annu. Rev. Condens. Matter Phys. **1**, 323 (2010).

[13] M. C. Marchetti, J.-F. Joanny, S. Ramaswamy, T. B. Liverpool, J. Prost, M. Rao, and R. A. Simha, Hydrodynamics of soft active matter, Rev. Mod. Phys. **85**, 1143 (2013).

[14] C. Bechinger, R. Di Leonardo, H. Löwen, C. Reichhardt, G. Volpe, and G. Volpe, Active particles in complex and crowded environments, Rev. Mod. Phys. **88**, 045006 (2016).

[15] S. Shankar, A. Souslov, M. J. Bowick, M. C. Marchetti, and V. Vitelli, Topological active matter, Nat. Rev. Phys. **4**, 380 (2022).

[16] H. Sompolinsky and I. Kanter, Temporal association in asymmetric neural networks, Phys. Rev. Lett. **57**, 2861 (1986).

[17] R. Mandal, R. Huang, M. Fruchart, P. G. Moerman, S. Vaikuntanathan, A. Murugan, and V. Vitelli, Learning dynamical behaviors in physical systems (2024), arXiv:2406.07856 [cond-mat.soft].

[18] C. Weis, M. Fruchart, R. Hanai, K. Kawagoe, P. B. Littlewood, and V. Vitelli, Coalescence of attractors: Exceptional points in non-linear dynamical systems (2022), arXiv:2207.11667 [nlin.CD].

[19] M. P. Zaletel, M. Lukin, C. Monroe, C. Nayak, F. Wilczek, and N. Y. Yao, Colloquium: Quantum and classical discrete time crystals, Rev. Mod. Phys. **95**, 031001 (2023).

[20] T. Liu, J.-Y. Ou, K. F. MacDonald, and N. I. Zheludev, Photonic metamaterial analogue of a continuous time crystal, Nat. Phys. **19**, 986 (2023).

[21] A. Marzo, A. Barnes, and B. W. Drinkwater, TinyLev: A multi-emitter single-axis acoustic levitator, Rev. Sci. Instr. **88**, 085105 (2017).

[22] M. A. Abdelaziz and D. G. Grier, Acoustokinetics: Crafting force landscapes from sound waves, Phys. Rev. Research **2**, 013172 (2020).

[23] M. Morrell and D. G. Grier, Acoustodynamic mass determination: Accounting for inertial effects in acoustic levitation of granular materials, Phys. Rev. E **108**, 064903 (2023).

[24] K. Yosioka and Y. Kawasima, Acoustic radiation pressure on a compressible sphere, Acta Acust. United Ac. **5**, 167 (1955).

[25] L. Gor'kov, On the forces acting on a small particle in an acoustical field in an ideal fluid, Sov. Phys. Dokl. **6**, 773 (1962).

[26] A. A. Doinikov, Acoustic radiation pressure on a rigid sphere in a viscous fluid, Proc. R. Soc. Lond. A **447**, 447–466 (1994).

[27] J. Horvath, Expanded polystyrene (EPS) geofoam: an introduction to material behavior, Geotext. Geomembr. **13**, 263 (1994).

[28] W. König, Hydrodynamisch-akustische Untersuchungen, Ann. Phys. **279**, 43 (1891).

[29] M. Settnes and H. Bruus, Forces acting on a small particle in an acoustical field in a viscous fluid, Phys. Rev. E **85**, 016327 (2012).

[30] G. T. Silva and H. Bruus, Acoustic interaction forces between small particles in an ideal fluid, Phys. Rev. E **90**, 063007 (2014).

[31] N. St. Clair, D. Davenport, A. D. Kim, and D. Kleckner, Dynamics of acoustically bound particles, Phys. Rev. Res. **5**, 013051 (2023).

[32] X. Zheng and R. E. Apfel, Acoustic interaction forces between two fluid spheres in an acoustic field, J. Acoust. Soc. Am. **97**, 2218 (1995).

[33] M. Morrell and D. G. Grier, Zenodo: Nonreciprocal wave-mediated interactions power classical a time crystal (2025), https://dx.doi.org/10.5281/zenodo.15237822.

[34] D. Lide, ed., *CRC Handbook of Chemistry and Physics*, 104th ed. (Taylor & Francis, 2017).

[35] M. R. Maxey and J. J. Riley, Equation of motion for a small rigid sphere in a nonuniform flow, Phys. Fluids **26**, 883 (1983).

[36] L. Landau and E. Lifshitz, *Fluid Mechanics*, Course of Theoretical Physics, Vol. 6 (Pergamon, 2013).

[37] E. Hinch, *Perturbation Methods*, Cambridge Texts in Applied Mathematics (Cambridge University Press, 1991).

[38] A. Shapere and F. Wilczek, Classical time crystals, Phys. Rev. Lett. **109**, 160402 (2012).

[39] N. Y. Yao, C. Nayak, L. Balents, and M. P. Zaletel, Classical discrete time crystals, Nat. Phys. **16**, 438 (2020).

[40] M. Fruchart, C. Scheibner, and V. Vitelli, Odd viscosity and odd elasticity, Annu. Rev. Condens. Matter Phys. **14**, 471 (2023).

[41] H. Taheri, A. B. Matsko, L. Maleki, and K. Sacha, All-optical dissipative discrete time crystals, Nat. Commun. **13**, 848 (2022).

[42] H. Keßler, P. Kongkhambut, C. Georges, L. Mathey, J. G. Cosme, and A. Hemmerich, Observation of a dissipative time crystal, Phys. Rev. Lett. **127**, 043602 (2021).

[43] K. Sacha and J. Zakrzewski, Time crystals: a review, Rep. Prog. Phys. **81**, 016401 (2017).

[44] M. Fruchart, R. Hanai, P. B. Littlewood, and V. Vitelli, Non-reciprocal phase transitions, Nature **592**, 363 (2021).

[45] M. Evers and R. Wittkowski, An active colloidal system showing parallels to a time crystal, Phys. Scr. **98**, 125240 (2023).

[46] Y. Avni, M. Fruchart, D. Martin, D. Seara, and V. Vitelli, Dynamical phase transitions in the nonreciprocal ising model, Phys. Rev. E **111**, 034124 (2025).