Aerodynamic Control of Laminar Separation on a Wall-Bounded Airfoil at Transitional Reynolds Numbers

A Preprint

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Abstract

Experiments were conducted in a low-turbulence wind tunnel to investigate the efficacy of localised acoustic forcing upon the dynamics and stability of the flow on a cambered, wall-bounded airfoil over a range of Reynolds numbers (Re) where the flow state can switch between two limits – a low-lift state (SI) where separation continues beyond the trailing edge and a high-lift state (SII) where the separated flow is closed off to form a laminar separation bubble. The switching between SI and SII can occur close to a critical angle of attack (α_{crit}) which varies with Re. The most effective forcing frequencies are found at a constant value of a rescaled Strouhal number, $\text{St}^* = \text{St}/\text{Re}^{1/2} = 0.027$, which indicates that though the primary unstable modes of the separated shear layer are of the inviscid, Kelvin-Helmholtz type, these modes are seeded by length scales that originate in the laminar (viscous) boundary layer. The most effective chordwise forcing location varies with $\text{St}/\text{Re}^{1/2}$ and incidence angle, α, and is always upstream of the separation point. Although the boundary layer flows are far from two-dimensional, forcing at a fixed chord location across all spanwise locations is effective in controlling the SI – SII transition. Strategies for active and passive feedback control are suggested.

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1 Introduction

1.1 Transitional Aerodynamics

As the chord-based Reynolds number, Re $= Uc/\nu$ (U is the flight speed, c is the chord length, and ν is the kinematic viscosity), of a flying vehicle falls below about 10^6, the aerodynamic properties become strongly influenced not only by the angle of incidence, α, but also Re itself (which is the source of only minor variations when Re $> 10^6$). In particular, in the range Re $= 10^4 - 10^5$, there is a sometimes delicate balance as the laminar boundary has relatively weak resistance to separation but can also reattach, in a time-averaged sense, thanks to the rapid development of shear-layer instabilities and their subsequent transition to turbulence. There is a strong dependence of performance metrics such as the lift to drag ratio, L/D, on whether the flow does or does not reattach; and there is considerable interest in the origin of separation as well as the development of initially small perturbations in the detached shear layer. When the shear layer instabilities are of sufficient amplitude, either in the form of spanwise coherent vortices or as turbulent fluctuations that bring high-speed fluid close to the wall, time-averaged reattachment occurs. The effective airfoil shape then includes the streamline curvature as the separated flow is closed off before the trailing edge, and this closed, recirculating region is termed a laminar separation bubble, or LSB [Tani, 1964]. As noted by Lissaman [1983], the dynamics of the LSB then dictate the aerodynamic performance of the wing section.

For a uniform, turbulence-free condition, far from boundary effects, the performance of a given wing section is a function of Re and α. The effects of Re variation were demonstrated by Zaman and McKinzie [1991], showing abrupt jumps in C_L as Re rose through the range of $5 - 7 \times 10^4$ for two airfoils (LRN-(1)-1007 and Wortmann FX 63-137), and more recently by Toppings and Yarusevych [2024] on a NACA 0018 across a range of $3° \leq \alpha \leq 9°$ by varying Re through $= 4.8 - 8.6 \times 10^4$ in a single time series. Doubling or halving of C_L was observed when Re was increased or decreased around a critical value where LSB formation would occur.

The variation with α was examined by Tank et al. [2021] on a NACA 65(1)412 at Re $= 2 \times 10^4$. A critical angle of attack (α_{crit}) was identified at which point the wing would experience an abrupt, non-linear jump in lift and drop in drag if α were increased by an increment as small as $0.5°$. Through time-averaged PIV fields, Tank et al. [2021] connected the discontinuity in the lift and drag polars to a change in flow state from one with laminar separation without reattachment (SI), to one with laminar separation with reattachment (SII), much as previously noted for the Eppler 387 by Yang and Spedding [2014]

In addition to the governing parameters Re and α, small variations in geometry and/or freestream conditions are known to influence the flow state. Both Jaroslawski et al. [2023] and Istvan et al. [2018] found that that higher freestream turbulence intensities promote transition, resulting in the formation and shrinking of LSBs. Such sensitivities likely are responsible for the difficulty in reconciling experimental and numerical results for ostensibly similar conditions [Selig, 1995, Yang and Spedding, 2013a, Tank et al., 2017].

A number of studies have investigated the mechanisms of transition in the shear layer originating from laminar separation [Burgmann et al., 2008, Jones et al., 2010, Rodriguez et al., 2013, Kurelek et al., 2021]. The boundary layer separates from the airfoil surface and initially remains laminar, creating a stagnant zone termed the dead-air region by Horton [1968]. As the separation line further diverges from the surface, reverse flow velocities increase and a strong shear layer develops. Exponential growth of disturbances causes the shear layer to destabilize into a two-dimensional instability similar to a Kelvin-Helmholtz (K-H) [Thomson, 1871, Helmholtz, 1868] instability, albeit with a nearby boundary. Diwan and Ramesh [2009] experimentally investigated laminar separation on a flat plate and found that the origin of perturbation growth was due to inflectional instabilities upstream of separation. Studies by Probsting and Yarusevych [2015] and Michelis et al. [2018] have also located the amplification of disturbances as occurring upstream of separation and confined to a select frequency range. The inflectional instabilities identified by Diwan and Ramesh [2009] were found to be convective in nature and the dominant mechanism for the growth of disturbances into the dead-air region. After detachment of the boundary layer and sufficient distancing of the separation streamline from the airfoil surface, the dominant amplification mechanism of disturbances becomes that of an inviscid shear layer instability, with higher growth rates. Similar findings by Yarusevych et al. [2006], Jones et al. [2008] and Boutilier and Yarusevych [2012] identified these K-H shear layer instabilities as the primary mechanism for disturbance growth through experiment and linear stability theory. In contrast to the convective instabilities discussed by Diwan and Ramesh [2009], studies by Alam and Sandham [2000] and Dellacasagrande et al. [2024] identified the onset of absolute instabilities for reverse flows of sufficient height or magnitude within the separated region. In either case, exponential growth of disturbances leads to two-dimensional roll-up vortices which initially remain coherent. Eventually, these roll-up vortices are destabilized by nonlinear interactions as they convect downstream and rapidly transition to turbulence.

Direct Numerical Simulations by Klose et al. [2025] detail the three-dimensional transition process for a NACA 65(1)412, noting the appearance and merging of braid vortices (also discussed by Ho and Huerre [1984] in relation to K-H instabilities) between successive roll-up vortices as the beginning of spanwise instabilities breaking up the spanwise

coherent rollers. The presence of Crow instabilities [Crow, 1970] brought on by trailing edge vortex interactions is also observed in the near-wake, further illustrating the range of physical mechanisms involved with transition over an airfoil. This brief survey gives context for the rich set of dynamics and flow structures present in transitional aerodynamic flows and the role their sensitivities play in the abrupt performance changes experienced by wings in this Re regime.

1.2 Tonal Noise Emission and The Acoustic Feedback Loop

The propensity of the shear layer to be affected by perturbations at specific frequencies, in conjunction with the unique possibility of tonal noise emission from airfoils at transitional Reynolds numbers, enables a self-sustaining process known as the acoustic feedback loop (AFL) [Longhouse, 1978, Arbey and Bataille, 1983, Golubev, 2021]. Essential to the acoustic feedback loop is the presence of airfoil tonal noise, most commonly at low α, when coherent vortex shedding at the trailing edge is spanwise uniform and periodic. Studies [Desquesnes et al., 2007, de Pando et al., 2014, Probsting et al., 2015] have investigated tonal noise emissions from airfoils and Golubev [2021] presents maps of airfoil noise emission for different Reynolds numbers and angles of attack. Probsting and Yarusevych [2015] showed that tonal noise will occur when coherent vortices are shed over the trailing edge of an airfoil from either the suction or pressure surface. When these vortices convect past the trailing edge, they will scatter and produce acoustic waves at the frequency of the vortex shedding. These waves will propagate in all directions, but importantly back upstream, where the most receptive regions are either at the maximum edge velocity or at the onset of instability in the laminar boundary layer, upstream of separation.

The presence of tonal noise is not required for the formation of an LSB, as pointed out by Jones et al. [2008], and conversely, the formation of an LSB does not guarantee tonal noise emission, as shown by Probsting and Yarusevych [2015]. Rather, the acoustic feedback loop will take place if the frequencies of the acoustic waves shed from the trailing edge are in the vicinity of unstable shear layer modes. When the acoustic waves interact with these unstable modes a phase lock between the acoustic wave frequency and shear layer shedding occurs [Probsting and Yarusevych, 2015, Thomareis and Papadakis, 2018].

1.3 Acoustic Forcing of Airfoils with Laminar Separation

The demonstrated sensitivity of the laminar boundary layer to small disturbances, and the known AFL mechanisms encourage controlled forcing methods through acoustic waves, where excitation sources may be external, or internal through miniature speakers embedded in the wing. A number of studies [Zaman and McKinzie, 1991, Yarusevych et al., 2006, Yang and Spedding, 2013b, 2014, Benton and Visbal, 2018, Celik et al., 2023, Coskun et al., 2024] have identified optimum forcing frequencies, or bands of effective frequencies, and have characterized the effect of varying amplitude and forcing location. Significant improvements in airfoil performance are possible with the implementation of acoustic forcing and marked changes in flow fields occur when excitation frequencies match instability frequencies as shown by Zaman and McKinzie [1991] and Yarusevych et al. [2007].

1.3.1 Optimal Frequency for Promoting Reattachment

Numerical and experimental studies have demonstrated that on airfoils with laminar separation, optimal excitation frequencies for promoting transition vary with Re, α, and airfoil surface curvature (i.e. the magnitude of the adverse pressure gradient at separation). Zaman and McKinzie [1991] first demonstrated that the optimal forcing frequency scaled with $U^{3/2}$ for a given airfoil chord length, and that the most effective frequencies occur when the parameter $\mathrm{St}/\mathrm{Re}^{1/2} = 0.02 - 0.03$. The authors also noted that for forcing to remain effective when the flow has switched to the reattached state, then it must impact the boundary layer itself, rather than exclusively a separated layer that may not exist. Zaman and McKinzie [1991] further demonstrated that the range of most effective frequencies shrinks with decreasing Re. Though the $1/\mathrm{Re}^{1/2}$ scaling suggests a dependence on laminar boundary layer dimensions, it was noted that significant amplification occurred in the separated shear layer. Based on this and the tonal noise and AFL discussion, an effective forcing causes a phase-lock between the forcing frequency and most amplified modes in the shear layer, via viscous length scales that are initially excited before separation [Yarusevych et al., 2005, 2007, Coskun et al., 2024].

1.3.2 Effect of Forcing Amplitude

Sweeps through varying excitation amplitudes, A_e, for known effective excitation frequencies have shown that a relatively small input energy can result in large performance improvements [Yarusevych et al., 2007, Yang and Spedding, 2014, Michelis et al., 2017, Coskun et al., 2024]. When forced at an excitation frequency, f_e, at or close to those that excite the K-H instabilities, $C_L(A_e)$ curves have a sigmoidal shape, so below a threshold A_e, there is only a small effect, and once the flow state has switched, further increases in A_e indicate a saturation in response.

1.3.3   Optimal Forcing Location

The most effective acoustic forcing location varies with Re and separation location x_s. Yeh et al. [2020] numerically investigated optimal actuator location for harmonic momentum-based forcing on a NACA0012 at $Re = 2 \times 10^5$ via resolvent analysis. The unforced flow included a separation bubble spanning $x/c = 0.022 - 0.08$. It was found that the optimal forcing location

$$x^* = \underset{x}{\operatorname{argmax}}\,(|\hat{f}|),$$

(where $|\hat{f}(x)|$ is the forced response mode magnitude) always occurred upstream of separation for forcing at a variety of frequencies. This finding is supported by the notion that perturbation growth begins upstream of separation, and is in agreement with previous numerical and experimental studies involving forcing of flows with laminar separation [Huang et al., 1988, Hsiao et al., 1990, Greenblatt and Wygnanski, 2000, Yeh and Taira, 2019]. Additionally, Yeh et al. [2020] explored optimal orientation of forcing, finding that a surface tangent direction was optimal for momentum flux forcing. At a higher Reynolds number of $Re = 3 \times 10^5$, experiments on a NACA $63_3 - 018$ airfoil by Hsiao et al. [1997] demonstrated delayed stall with leading edge forcing and showed most effective control when the shear-layer shedding "locked in" with acoustic forcing and when forcing occurred at the separation point. Yang and Spedding [2014] showed that improvements in $\Delta L/D$ for an Eppler 387 at $Re = 6 \times 10^4$ were independent of actuator location, but the range of frequencies which could yield an increase in L/D shrank as the actuator location moved from $x/c = 0.1$ to 0.6.

1.4   Scope and Objectives

The susceptibility of transitional flows to specific perturbation, combined with the resulting improvements in wing performance upon the promotion of transition and reattachment, renders them suitable, in principle, for control. The aim of this investigation is to expand on the current literature by experimentally investigating chordwise actuator location (x_e) on a finite wall-bounded wing across a parameter range of Re and α as well as excitation frequency (f_e) and amplitude (A_e). This will be accomplished through direct force measurements and flow field measurements on a NACA 65(1)412 airfoil in the Dryden Wind Tunnel (DWT). Comparisons with numerical and experimental results will be made when possible with the objective of identifying consistent flow structures and optimal actuator placement.

Figure 1 shows a road map for addressing the scope of this study. First, in section 3.1 characterization of the baseline flow is achieved through sweeps of α at $Re = 2, 4, 6, 8 \times 10^4$. α_{crit} is identified for each Re, where the flow switches state between separation without time-averaged reattachment and an LSB. This is followed by forced experiments starting in section 3.2. Initial frequency sweeps at the forward-most forcing location on the airfoil are conducted for each Re at α_{crit} and optimal forcing frequencies (St_e^*) are observed, with respect to improvement in airfoil performance (L/D). In the subsequent section (3.3) St_e^* optimality is demonstrated across streamwise forcing location (x_e) and angle of attack (α). In addition, section 3.3 assesses forcing effectiveness as a function x_e and identifies optimal forcing locations x_e^*. In section 3.4 the α dependence of optimal parameters St_e^* and x_e^* are investigated by sweeping through x_e and α at each Re while fixing the forcing frequency at St_e^*. Finally, follow-up experiments are presented in section 3.5. These results—and their implications and limitations—are discussed in section 4.

Figure 1: Experimental procedure for systematic investigation of parameter space.

Figure 2: Dryden Wind Tunnel Schematic.

2 Experimental Methods

Experiments were conducted in the Dryden Wind Tunnel (DWT). The DWT is a recirculating atmospheric wind tunnel with three octagonal test sections in series. The wall to wall width of each test section is 1.37 m and a schematic of the wind tunnel side-view is shown in figure 2. The wind tunnel has 9 mesh screens to break down turbulence before entering the settling chamber, with a 7:1 contraction ratio from the settling chamber to the first test section. The turbulence intensity has been measured by Tank [2018] to be $T \leq 0.035\%$ for $10 \leq f \leq 200$ Hz in the velocity range $4 \leq U \leq 20$ m/s.

2.1 Airfoil Model

A NACA 65(1)412 profile was used to generate two models, both with a chord length $c = 200$ mm and, span $b = 600$ mm. The first model was a one-piece PLA construction and the second was a two-piece PLA model created for internal acoustic excitation. The bottom pressure side carriage of the two-piece model contained channels and cavities in which speakers and associated wiring could be placed (Figure 3). The top piece was a thin shell suction side that slid onto the carriage and could be easily modified and replaced. Each model was sanded to a P1000 grit finish, alternately sanding and painting with a sandable filling primer to ensure a smooth surface.

2.2 Experimental Configuration

The models were mounted vertically between two end walls with dimensions 1200 x 600 x 4.8 mm^3. The top end wall was transparent to allow for PIV and flow visualizations. The bottom end wall was black to reduce light reflections in the wind tunnel. The leading and trailing edges of the end walls were tapered to a right–triangular edge with an angle of 60°. The coordinate system had its origin at the leading edge of the airfoil with x and y axes parallel and perpendicular to the freestream direction and the z axis traversing the span from end wall to end wall (Figure 4).

The wing was supported by a sting which was mounted to an ATI Gamma force balance for direct measurements of aerodynamic forces. A shroud was placed around the section of the sting exposed to the freestream in order to isolate the forces experienced by the wing. The force balance was mounted to a Parker PM-DDD12DNOH rotary table allowing for α sweeps. The assembled experimental setup is shown in figure 5.

The airfoil α was aligned with the freestream by setting up a calibration plane parallel with the freestream using a Bosch GLL 55 self-leveling cross-line laser. A PLA negative contour of the airfoil was mounted near mid-span on the airfoil such that the mounting screws intersected the laser sheet. Using a camera to locate the intersection of the laser plane and the screws, the airfoil was rotated until the laser intersected both screws at the same location. This location was considered $\alpha = 0°$. As the uncertainty in laser plane–screw intersection was on the order of 0.1 mm, the uncertainty in the $\alpha = 0°$ designation is on the order of 0.03°

The mean reference speed, U, was measured with a pitot tube, mounted 1 m upstream of the wing, and connected to a 0.5INCH-D2-4V-MINI differential pressure sensor.

Figure 3: Two-piece NACA 65(1)412 model. (Top Left): Isometric view of model with suction surface partially mounted with inside of bottom carriage visible. (Bottom Left): Cross section view with speaker cavities and wire channel visible. (Right): Top view of suction surface with speaker holes highlighted.

Figure 4: Model-based coordinate system and geometry of wall-bounded airfoil.

NACA 65(1)412

Figure 5: The NACA 65(1)412 model and force balance configuration.

Range				Resolution			
$F_{A,N}$	F_z	$\tau_{A,N}$	τ_z	$F_{A,N}$	F_z	$\tau_{A,N}$	τ_z
32 N	100 N	2.5 N-m	2.5 N-m	6.25 mN	12.5 mN	5 mN-m	5 mN-m

Table 1: Resolution and range of ATI Gamma Force / Torque sensor.

2.3 Force Balance

An ATI Gamma force balance with an SI-32-2.5 calibration was used to capture the forces and torques on the airfoil. The range and resolution of the balance are given in table 1 where subscripts A and N represent the axial and normal directions based on the chord aligned coordinate system. Prior to experimentation, estimates of lower limiting forces on the wing were made from previous data [Tank, 2018], and estimated to be 5 mN for Re $= 2 \times 10^4$ and 2.9 N for Re $= 8 \times 10^4$. The minimum force estimate is just below the resolution of the force balance resulting in a signal to noise ratio close to 1 for drag data near $\alpha = 0°$ at Re $= 2 \times 10^4$. To mitigate the limitations of the force balance at such low speeds, statistics were compiled through repeated experiments using different mountings and models. The results were averaged and uncertainties were reported as test-to-test variation about the mean.

The measured normal and axial forces were rotated to conventional lift and drag components by

$$L = F_N \cos(\alpha - \gamma) - F_A \sin(\alpha - \gamma),$$
$$D = F_A \cos(\alpha - \gamma) + F_N \sin(\alpha - \gamma),$$

where L is lift, D is drag and γ is the alignment offset. Weight tares were conducted before the experiments to remove the components of the force balance signal due to the weight of the wing. These were applied before the forces were transformed.

To compensate for any misalignment between the force balance axes and airfoil axes, a pure drag force calibration was applied to the strut after mounting and aligning α. The force was applied by attaching a string to the strut with the other end of the string hung over a pulley and attached to a weight. The string and pulley were aligned to the center line of the wind tunnel with the Bosch GLL 55 for a pure drag force. Force measurements were taken at one degree

increments for a hysteresis loop from $-5° \leq \alpha \leq 5°$. The difference between the angle measured by the force balance and α_0 was computed using

$$\gamma = \alpha_0 - \tan\left(\frac{F_N}{F_A}\right), \tag{1}$$

and the value was averaged for the four test points nominally at $\alpha = 0°$.

Gaps between the airfoil and end walls were necessary to isolate the force measurements on the airfoil. These gaps were measured at the leading and trailing edge for each experiment. Typical end wall gaps were ≤ 3 mm ($\leq 0.005b$) in accordance with Barlow et al. [1999]. Yaw estimates were computed from the end wall measurements and were $\leq 0.5°$ for all experiments. Roll estimates were made by aligning the Bosch GLL 55 with the trailing edge at $z = b$ and measuring its deviation from vertical at $z = 0$. The roll was $\leq 1.4°$ for all experiments.

Force measurements were taken with a sampling frequency of $f = 2000$ Hz, equating to $\Delta t U/c \leq 0.015$. α sweeps at fixed Reynolds numbers were conducted by setting the calculated wind speed for a given Re based on measured temperature, pressure, and relative humidity, waiting 120 s for the wind tunnel to reach a steady state, and then acquiring data. Data were acquired for 10 s ($\geq 75c/U$) at each α with 20 s ($\geq 150c/U$) settling times for the flow and mechanical setup to reach steady state after moving to a new α; for runs where α did not vary, such as St_e sweeps, a 5 s ($\geq 35c/U$) settling time was used instead. α sweeps were conducted as hysteresis loops with α starting at $\alpha = 0°$ incrementing up to $\alpha_{\max}$, then down to $\alpha_{\min}$, and finally back to $\alpha = 0°$.

The force measurements were normalized by the dynamic pressure q and the wing planform area $S = bc$:

$$C_L = \frac{L}{qS},$$
$$C_D = \frac{D}{qS}.$$

2.4   Acoustic Forcing and Measurements

Internal acoustic forcing experiments were conducted with speakers mounted inside the wing at a single chordwise location and nine uniformly spaced spanwise locations. The excitation voltage, $V(t)$, was

$$V(t) = A_e \sin(2\pi f_e t), \tag{2}$$

where f_e is the excitation frequency and A_e is the amplitude. During experiments at a given chordwise location, the remaining unused cavities were filled with PLA printed blocks to prevent acoustic resonances within the empty cavities. The unused speaker holes on the suction surface were filled with putty and lightly sanded to minimize their surface roughness impact. Figure 3 shows the different chordwise ($x_e/c = 0.1 - 0.6$, $\Delta x_e/c = 0.1$) and spanwise ($z_e/b = 0.1 - 0.9$, $\Delta z_e/b = 0.1$) excitation locations. Certain follow-up experiments were conducted with a higher spanwise density of speaker locations ($z_e/b = 0.025 - 0.975$, $\Delta z_e/b = 0.025$) at the foremost chordwise position, $x_e/c = 0.1$. The speakers were UT-P2020 MEMS speakers with a nominal frequency range from 20 Hz to 20 kHz. The speakers were wired in parallel to ensure the signals were in phase and the speakers were driven by a UA-R3010 amplifier and 33210A Agilent function generator. The wiring for the speakers was routed out of the wing at the bottom end wall and fixed to the sting so the entire setup moved as one rigid body.

The UA-R3010 is a linear amplifier for piezoelectric speakers with a 23.5 dB gain for a frequency range from 20 Hz-80 kHz. Its nominal 23.5 dB gain was found to drop off significantly at the lower frequencies ($f_e \leq 200$), as shown in figure 6. A lookup table was created with the experimental frequency response data, and to compensate for the drop off, higher input voltages were used for lower frequencies to ensure the speakers always received a signal of $A_e = 28.1$ V peak-peak (V_{PP}) ± 0.1 V.

The sound pressure level (SPL) of the speakers at $x_e/c = 0.3$ at an input voltage of $A = 28.1$ V_{PP} was measured with a 4954-B 1/4" B&K free field microphone mounted to a traverse. The test was conducted with the microphone mounted normal to the speaker hole location at a distance of ≤ 0.1 mm to the suction surface. The microphone was traversed through different spanwise locations. At each location, the speaker was swept through 28 logarithmically spaced points from 20 Hz to 500 Hz, and a Goertzel filter was applied to identify the measured SPL level of the excitation. The results are shown in figure 16.

Force measurements for a given chordwise actuator location began with an unforced α sweep at the same Re to check the effect of speaker holes themselves, and to verify agreement with previous unforced, baseline data. Next, α was set to α_{crit} for the test Re and a discrete frequency sweep from $2.7 \leq \mathrm{St}_e \leq 15$ was conducted, where $\mathrm{St}_e = f_e c/U$ is the excitation Strouhal number. Finally, a forced α sweep with $\mathrm{St}_e = \mathrm{St}_e^*$ was carried out, where St_e^* is the optimal forcing

Figure 6: Frequency response of UA-R3010 internal excitation amplifier

frequency at the given Re. The process was repeated for Re $= 2, 4, 6, 8 \times 10^4$ and then the location of excitation, x_e, was changed. x_e was varied from $x/c = 0.1 - 0.6$. St_e^* was fixed from the frequency sweep conducted at $x/c = 0.1$. The optimal, or most effective, excitation frequency was defined as St_e at which the maximum L/D occurred.

2.5   Particle Image Velocimetry

PIV images were acquired by seeding the wind tunnel with smoke, illuminating the smoke particles with a laser, and acquiring successive image pairs of the particles with a camera. The smoke was injected into the flow in the diffuser of the wind tunnel with a Colt 4 smoke machine. The smoke had a typical particle size of $0.2 - 0.3$ microns resulting in a settling velocity ≈ 8 mm/hr. A Quantel EverGreen double-pulsed Nd:YAG laser was used in conjunction with two plano-concave lenses in series to produce a laser sheet in the x-y plane and illuminate the particles. The x-y plane was centered above the airfoil suction surface. The laser could produce two 200 mJ pulses with wavelengths of 532 nm at a 15 Hz rate with time between pulses down to 1 μs. An Imager sCMOS camera with a spatial resolution of 2560 x 2160 pixels and a maximum frame rate of 50 FPS was used to acquire the images. A 50 mm Nikon lens focused at ≈ 0.8 m with an aperture setting of $f = 2.8$ was mounted on the camera and captured a field of view from the airfoil leading edge to the trailing edge. LaVision's DaVis software was used to sync the laser and camera for data acquisition, process the images with PIV algorithms to produce instantaneous vector fields, and produce uncertainty estimates. Figure 7 shows a representative instantaneous uncertainty field for the PIV measurements. An initial correlation box size of 48x48 pixels with 50% overlap was reduced to a final correlation box size of 16x16 pixels, with a multi pass FFT window deformation PIV algorithm. This produced vector fields with spatial resolution 0.79 mm ($0.0039c$) in the x and y directions.

The maximum temporal rate of data capture was limited by the laser at 15 Hz making the results non-time resolved since $U\Delta t/c \geq 0.5$ and a convergence study was conducted to determine the number of image pairs necessary for the flow to produce statistically stationary behavior. Two criteria were used: the average vector magnitudes were required to change by $\max(\Delta|\vec{u}|/U) < 0.1$ and the vector orientations were required to change by $\max(\Delta\theta) < 1°$ for the addition of one instantaneous vector field to the statistical population. 150 images were determined to be sufficient. Figure 8 shows the difference in magnitude and orientation of the time averaged flow for populations of 149 and 150 images. The non-time resolved nature of the PIV requires the use of ensemble averages which are denoted by an over-bar (e.g. $\bar{u}$)

10

Figure 7: Representative time averaged velocity field (left) and its corresponding uncertainty distribution (right) for DWT PIV.

Figure 8: Difference in vector magnitude (left) and orientation (right) for time averaged PIV fields with 150 images vs 149 images.

PIV measurements for the one–piece model were collected during the force measurements at $\alpha = 0°, 4°, \alpha_{\text{crit}}$, and $\alpha_{\text{crit}} + 1°$, for α sweeps at all Re. PIV measurements for the two-piece model with internal excitation were captured at α_{crit} for both the forced and unforced cases. Force measurements were taken in tandem to ensure agreement with previous data. Flow field data were taken at mid-span ($z/b = 0.5$).

To relate acoustic excitation location (x_e) to boundary layer parameters, estimates of the separation location were computed. The raw PIV spatial resolution was insufficient to accurately identify the separation location as given by the Prandtl separation criterion. Hence, vorticity contours of $\bar{\omega}_z c/U$ were examined and the shear layer was identified at each x/c location downstream of separation by

$$y_s = \underset{y}{\text{argmax}} |\overline{\omega}_z|. \tag{3}$$

A cubic polynomial was fit to the points in the shear layer and then extrapolated to the airfoil surface for separation point estimates. Figure 20 shows the $\bar{\omega}_z$ contours with the cubic polynomial and separation point estimate.

Figure 9: (Top) Unforced baseline lift and drag plots for one–piece PLA model Re $= 4 \times 10^4$. Red circles denote values corresponding to PIV fields. Uncertainties are the standard deviation of temporal fluctuations about the mean. (Bottom) Corresponding ensemble averaged PIV $\bar{u}$ velocity fields with streamlines for varying α.

3 Results

3.1 Baseline Measurements

Flow fields around the one–piece airfoil at Re $= 4 \times 10^4$ are shown in figure 9 to illustrate the abrupt transitions in flow state, together with consequences in the time-averaged lift and drag forces. Starting at $\alpha = 0°$ and increasing α, C_L linearly increases until $\alpha = 5°$. The respective PIV contours show a separation region near the trailing edge which grows and moves towards the leading edge as α increases. After $\alpha = 5°$, the C_L curve begins to diverge from the $2\pi\alpha$ line with the lift slope, $C_{L\alpha}$, degrading. At $\alpha = 7°$, the separated region reaches its maximum height, and extends well beyond the trailing edge. The lower curvature of the streamlines is associated with the reduction in $C_{L\alpha}$. $\alpha = 7°$ is termed α_{crit} [Tank et al., 2021] denoting that the flow experiences a state change when α is increased beyond α_{crit}. At $\alpha = 8°$, $C_L(\alpha)$ has increased abruptly, accompanied by a drop in C_D. C_L at $\alpha = 8°$ outperforms two-dimensional airfoil theory because the streamlines experience an increase in curvature over the suction surface, as they round the closed separation bubble, increasing circulation, as would occur with a more highly–cambered profile shape. Post-$\alpha = 8°$, both C_L and C_D increase with α, but on different curves than the pre-α_{crit} state. The performance change in the force measurements after α_{crit} is connected to the collapse of the separated region and development of an LSB in the PIV fields.

Figure 10 shows the nonlinearities in $C_L(\alpha)$ gradually straightening out as Re increases. The $C_L(\alpha)$ curves shift upwards (so at any fixed α, C_L increases), and the increase in lift slope, $C_{L\alpha}$ with Re, and decrease in α_{0L} (zero–lift angle of attack) are as noted in Tank et al. [2021]. With the reduced influence of viscosity, C_D at any given α also drops too. The PIV contours demonstrate the thinning of the boundary layer for a fixed α as Re is increased. Notably, for Re $= 8 \times 10^4$ the separated region is very thin in the PIV contour and does not push the curving of the streamlines into the airfoil wake. This is likely true for all α at Re $= 8 \times 10^4$ and explains the lack of distinct α_{crit}. At Re $= 2 \times 10^4$ the C_L curve begins to decrease after the switch in flow state and diverges from the collapsed results of the higher Re cases. Previously, it has been shown [Klewicki et al., 2025] there is an onset of turbulent separation downstream of the LSB which explains the decrease in C_L.

Figure 10: (Top) Unforced baseline lift and drag plots for one–piece PLA model Re $= 2 - 8 \times 10^4$. Red circles denote values corresponding to PIV fields. Uncertainties are the standard deviation of temporal fluctuations about the mean. (Bottom) Corresponding ensemble averaged PIV $\overline{u}$ velocity fields with streamlines for fixed α.

The contours and streamlines in figure 11 at $\alpha = 0°$ demonstrate the boundary layer thinning as Re is increased (top to bottom) with a shrinking separated region near the trailing edge. At $\alpha = \alpha_{\text{crit}}$ this becomes more apparent as the separation regions reach their maximum extent for each Re and finally at $\alpha = \alpha_{\text{crit}} + 1°$ the switch from open separation to the formation of a closed LSB is observed. The streamlines for the Re $= 2 \times 10^4$ case (top row) show a separation bubble which has formed on the first half of the chord. This is distinct from the results at higher Re which show LSBs initially forming closer to the trailing edge on the downstream half of the chord. The lower Re $= 2 \times 10^4$ increases the stability of the separated shear layer which in turn requires more development in the streamwise direction for vortex roll up to occur, leading to a higher α_{crit}. This trend should hold in general regardless of airfoil contour. For this particular airfoil, the maximum thickness at $x/c \approx 0.4$ allows for a relatively large leading edge LSB at Re $= 2 \times 10^4$, while airfoils with maximum thicknesses closer to the leading edge may lead to a more conventional bifurcation of large LSBs on the aft part of the chord and small leading edge LSBs.

To check the possible influence of small geometric disparities in different fabrication strategies, Figure 12 compares the force measurements of the one- and two-piece airfoil models. The principal differences lie in C_D, which is different beyond experimental uncertainty at low Re and α, and also different for the highest Re and α below α_{crit}. Given that this flow is known to be highly sensitive to small variations in geometry, the relatively small differences observed here are likely due to slightly higher effective surface roughness on the two-piece construction. Otherwise, the small influence allows us to continue while using the two-piece airfoil as a baseline case for remaining experiments and the α_{crit} for each Re will be identified based on the two-piece wing data in figure 15.

As part of this collaborative work, lift and drag values from three- and two-dimensional DNS were computed for Re $= 2 \times 10^4$. The unforced experiments for each model were compared with previous experimental work [Tank et al., 2021] and corresponding DNS [Klose et al., 2025] in figure 13. Good agreement is shown between experiment and DNS for $\alpha < 7°$ and $\alpha > 10°$. The DNS identifies $\alpha_{\text{crit}} = 7°$ while experiments identify $\alpha_{\text{crit}} = 9°$, leading to major discrepancies for lift and drag over $7° < \alpha < 9°$. The current experiments showed good agreement with previous experiments, demonstrating similar α_{crit} values. $|C_L|$ from Tank et al. [2021] is slightly higher than from the current experiments and may be due to the difference in aspect ratio $AR = 3$ (current) vs $AR = 12.7$, a difference that Tank et al. [2021] also showed to be responsible for a reduction in α_{crit} of $0.5°$. Plotting only the data at Re $= 2 \times 10^4$ allows for

13

Figure 11: Ensemble averaged PIV $\overline{u}$ velocity fields with streamlines for $\alpha = 0, \alpha_{\mathrm{crit}}, \alpha_{\mathrm{crit}} + 1°$ (Left to Right) and Re $= 2, 4, 6, 8 \times 10^4$ (Top to Bottom).

Figure 12: Unforced baseline lift and drag data for one–piece (•) and two-piece (▲) models. The two-piece model was configured with no speaker holes in the suction surface. Uncertainties are the standard deviation of test-test variation of the mean. The theoretical $2\pi\alpha$ line intersects the origin.

Figure 13: Lift and drag plots at Re $= 2 \times 10^4$ for different models compared with previous experimental and numerical results. Uncertainties for experiment are the standard deviation of test-test fluctuations of the mean. Uncertainties for simulation are the standard deviation of temporal fluctuations about the mean.

Figure 14: $\Delta L/D$ relative to two–piece model with smooth suction surface (baseline), for the suction surface with spanwise speaker holes (no acoustic excitation) at different chordwise locations (x_{holes}). Re $= 2, 4, 6, 8 \times 10^3$ from top left to bottom right.

a C_L discontinuity at negative α to be seen near $\alpha = -3°$. This is likely due to separation dynamics occurring on the pressure side of the airfoil and does not occur for Re $\geq 6 \times 10^4$.

To evaluate the aerodynamic effect of introducing orifices for acoustic excitation on the suction surface, a comparison of the baseline two-piece airfoil measurements and the two-piece airfoil with holes at different chordwise locations is shown in figure 14. For these runs, inactive speakers were placed inside their cavities to minimize Helmholtz resonance effects. At Re $= 2 \times 10^4$ L/D is most affected by $x_{\text{holes}}/c = 0.4$ increasing L/D for $2° \leq \alpha \leq 9°$. For Re $\geq 4 \times 10^4$ locations $x_{\text{holes}}/c = 0.2 - 0.5$ introduce moderate increases in L/D with the effect becoming more pronounced as Re increases. Locations of x_{holes} closer to the leading edge have effects which persist over larger ranges of α and increases in L/D experience sharp drop offs at Re $= 6$ & 8×10^4 for $x_{\text{holes}}/c \geq 0.3$ & 0.4 respectively.

3.2 Forcing Frequency Identification

Sweeps through different forcing frequencies at α_{crit} for each Re are shown on the right in figure 15. For reference, the baseline α sweeps are shown on the left with α_{crit} identified. The sweeps were conducted at α_{crit} where the separated region is the largest, making it most susceptible to forcing. To isolate the effects of forcing, the forcing location $x_e/c = 0.1$ was selected for the initial frequency sweeps since the presence of the speaker holes had little to no effect on the airfoil performance at this location, as shown by figure 14. The frequency sweeps for Re $\geq 4 \times 10^4$ generate

Figure 15: α sweeps of baseline configuration with α_{crit} identified by red circles for each Re (left). Frequency sweeps at α_{crit} with $x_e/c = 0.1$ and St_e^* identified by orange circles for each Re (right). $\mathrm{St}_e = 0$ data point corresponds to unforced measurement at beginning of frequency sweep.

Figure 16: Frequency response of internal excitation with wind tunnel off; measured at a distance of ≤ 0.1 mm to the suction surface at $x/c = 0.3$, with speakers mounted at $x_e/c = 0.3$.

concave curves of $L/D(\mathrm{St})$, while the frequency sweep at $\mathrm{Re} = 2 \times 10^4$ is unaffected by the forcing. The optimal forcing frequency was identified as

$$\mathrm{St}_e^* = \underset{\mathrm{St}_e}{\operatorname{argmax}} \frac{L}{D}.$$

The UT-P2020 MEMS speakers selected for forcing had a nominally flat amplitude response for frequencies from $f = 20 - 700$ Hz according to their data sheet. However, acoustic measurements of the speakers from figure 16 showed decreasing SPL with a decrease in f. The contour yields ridges of high SPL at the speaker locations every $0.1b$ in the spanwise direction. The highest ridge is at the center location due to the constructive effects of the sound waves. The frequency axis shows diminishing SPL as the frequency decreases, and below 60 Hz the microphone self–noise dominates. St_e^* for each Re is located on the contour and shows that the relative ineffectiveness of forcing for $\mathrm{Re} = 2 \times 10^4$ is likely more related to a reduced forcing amplitude than an intrinsic property of the flow. For the following plots only $\mathrm{Re} = 4 - 8 \times 10^4$ will be considered.

Taking the data from the frequency sweeps and applying the scaling proposed by Zaman and McKinzie [1991] the optimal rescaled forcing frequencies, St_e^*, have a constant value $\mathrm{St}_e/\mathrm{Re}^{1/2} \approx 0.027$. This value agrees with power spectral density measurements taken in the shear layer by Klewicki et al. [2025], which came from the same airfoil at $\alpha = 6°$ and the same Reynolds numbers. This indicates that the forcing is interacting with shear layer vortex shedding, perturbing the system with frequencies near unstable shear layer modes. The proximity of the forcing to the most

Figure 17: L/D from frequency sweeps with $St/Re^{1/2}$ scaling [Zaman and McKinzie, 1991]. Uncertainties are the standard deviation of temporal fluctuations about the mean.

Figure 18: Schematic of laminar separation bubble with shear layer profile.

unstable shear layer modes will dictate temporal and spatial growth rates changing the size and shape of any resulting LSBs. Previous studies [Watmuff, 1999, Yarusevych et al., 2009, Diwan and Ramesh, 2009, Marxen et al., 2012, Dellacasagrande et al., 2024] have pointed to Kelvin-Helmholtz type instabilities as the primary instability. Physical arguments can be made to relate the $St_e/Re^{1/2} = 0.02 - 0.03$ values for optimal forcing cited by Zaman and McKinzie [1991] to Kelvin-Helmholtz instabilities occurring in the laminar separation shear layer. If the most amplified K-H frequency of $f_{KH} = 0.032\bar{U}/\theta_{SL}$ from Ho and Huerre [1984] is used as the excitation frequency, where $\bar{U} = (U_1+U_2)/2$ is the mean velocity of the shear layer, and θ_{SL} is the momentum thickness of the shear layer, it is possible to approximate the values from experiment. Assuming $U_2 \approx 0$, $U_1 = U$, using the flat plate Blasius boundary layer thickness estimate $\theta_{SL} \approx \theta_{FP} = 0.664 x_s/Re_{x,s}^{1/2}$ for the shear layer momentum thickness, and defining the separation point as some fraction of the chord $x_s = c/n$, where $1 \leq n$, then

$$f_{KH} = 0.032\frac{\bar{U}}{\theta_{SL}} = 0.032\frac{U}{2}\frac{\sqrt{Re_{x,s}}}{(0.664)x_s} = 0.032\frac{U}{2}\frac{\sqrt{Re}\sqrt{n}}{(0.664)c},$$

$$\frac{St_e}{Re} = \frac{f_e c}{U\sqrt{Re}} = \frac{f_{KH}c}{U\sqrt{Re}} = \frac{0.032\sqrt{n}}{2(0.664)} = 0.024\sqrt{n}.$$

A naive assumption of $n = 1$, where the characteristic length (x_s) of the separated shear layer momentum thickness (θ_{SL}) equals the chord length, yields $St/Re^{1/2} = 0.024$, similar to the value of 0.027 from this study and 0.025 from Zaman and McKinzie [1991]. A more empirically based approximation from table 2 of $n = 2$–8, where separation is between eighth-chord and mid-chord gives values of $St_e/Re^{1/2} = 0.034 - 0.067$. The lower end of this range agrees

Figure 19: C_L and C_D from frequency sweeps with $St/\sqrt{Re}$ scaling [Zaman and McKinzie, 1991]. Uncertainties are the standard deviation of temporal fluctuations about the mean.

α	4°	5°	6°	7°	8°	9°	10°
Experiment at α_{crit}							
x_s/c	0.50	0.45	-	0.26	-	0.156	-
DNS at Re = 2×10^4							
x_s/c	0.49	-	0.4	0.26	0.014	-	0.012

Table 2: Separation point locations from DNS [Klose et al., 2025] and experiment. $\alpha_{\text{crit}} = 4°, 5°, 7°, 9°$ for experiment correspond to Re = $8, 6, 4, 2 \times 10^4$ respectively.

with data from Michelis et al. [2018], Yang and Spedding [2014], and Yarusevych et al. [2007]. The upper end of this estimate is likely not realized as St_e saturates at the K-H frequency when separation takes place at smaller fractions of the chord (higher α), causing the shear layer to diverge more quickly from the suction surface. Zaman and McKinzie [1991] provide evidence of this with $St_e^*/Re^{1/2} = 0.22 - 0.28$ over $\alpha = 0° - 6.5°$ for the sharp–nosed LRN airfoil. Overall, this analysis is similar that of Watmuff [1999] which compares experimental vortex shedding frequency of a flat plate LSB to canonical linear stability analysis by Monkewitz and Huerre [1982] and two-dimensional vortex shedding work by Pauley et al. [1990]. Although the K-H instability is an inviscid instability, for the case of wall-bounded laminar separation, the initial momentum thickness of the shear layer is set by the laminar boundary layer. A cartoon is shown for an airfoil in figure 18 and the scaling leads to the collapse of optimal forcing frequencies (St_e^*) for this study. Most of the studies discussed here fall into the values $n = 1 - 2$ indicating that a length scale between mid-chord and the full chord length will result in an estimate close to whats been observed: $St_e^*/Re^{1/2} = 0.24 - 0.34$.

Further interrogation of the lift and drag in figure 19 shows the lift exhibits a plateau at the center of which the optimal frequency occurs, while drag exhibits convex curves driving the identification of global optimum values.

3.3 Forcing Location Trends

Separation point estimates are shown in figure 20 and are plotted in the subsequent contours of x_e vs. St_e. The figure shows that the higher α_{crit}, at lower Re, corresponds to earlier separation and larger separated regions.

A comparison of the separation points identified by computations and experiment is shown in table 2. For $\alpha \leq 7°$ the separation points show good agreement between DNS and experiment. For $\alpha > 7°$, the DNS at fixed Re = 2×10^4 shows the result when α is significantly greater than α_{crit} and the separation point has reached its foremost location.

The frequency sweeps from the previous section were carried out at the subsequent chordwise locations $x/c = 0.2 - 0.6$ and contours comparing the performance with the smooth suction surface (baseline) and the suction surface with speaker holes (no acoustic forcing) are shown in figures 21 & 22. Increases in L/D up to 10 were achieved relative to the baseline and figure 22 shows that acoustic excitation enhanced performance beyond the improved L/D from surface roughness introduced by the speaker holes present on the suction surface. The optimal frequencies (St_e^*) are indicated in both figures for their respective Re and show the best performance improvement at each forcing location. This demonstrates that the optimal forcing frequency is independent of forcing location. While some L/D improvement was still observed downstream of the separation line the locations of largest $\Delta L/D$ for both sets of contours all occurred with forcing upstream of separation.

Figure 20: Estimates of separation point. Contours of $\bar{\omega}c/U$ with cubic polynomial fit to shear layer and extrapolated to airfoil surface. ($Re = 2, 4, 6, 8 \times 10^3$ from top left to bottom right.)

Figure 21: Contours of $\Delta L/D$ relative to smooth baseline from frequency sweeps at different forcing locations x_e. Data points at St_e^* for each x_e denoted by colored circles. Black dashed line indicates separation point estimate x_s. $Re = 4, 6, 8 \times 10^4$ at corresponding $\alpha_{\text{crit}} = 7°, 5°, 4°$ left to right

Figure 22: Contours of $\Delta L/D$ relative to suction surface with x_{holes}, from frequency sweeps at different forcing locations x_e. Data points at St_e^* for each x_e denoted by colored circles. Black dashed line indicates separation point estimate x_s. $\text{Re} = 4, 6, 8 \times 10^4$ at corresponding $\alpha_{\text{crit}} = 7°, 5°, 4°$ left to right

Figure 23: $\%\Delta L/D$ relative to baseline for data points at St_e^* at each x_e from figure 21 (left). Difference between separation (x_s) location and optimal forcing location $x_e^* = \underset{x_e}{\text{argmax}} \frac{L}{D}$ as a function of Re (right).

The individual Re curves in figure 23 show an initial increase in the effectiveness of forcing as x_e is moved downstream from $x_e/c = 0.1$, followed by a steep fall-off after reaching their maximum. In the case of $\text{Re} = 4 \times 10^4$, a reduction in L/D occurs for $x_e/c \geq 0.4$. The x_e location with the maximum increase in L/D shifts downstream as Re increases. The maximum L/D for $\text{Re} = 4 \times 10^4$ occurs at $x_e/c = 0.2$, with near parity between $x_e/c = 0.2$ and 0.3 at $\text{Re} = 6 \times 10^4$, and a maximum at $x_e/c = 0.3$ for $\text{Re} = 8 \times 10^4$. These optimal locations x_e^* for $\text{St}_e = \text{St}_e^*$, where

$$x_e^* = \underset{x_e}{\text{argmax}} \frac{L}{D},$$

can be compared with the estimated separation location x_s, and the distance upstream of separation at which x_e^* occurs is shown on the right in figure 23. The location of x_e^* moves further upstream of x_s as x_e^* increased, perhaps related to the increased amplitude of forcing at higher St_e, earlier onset of instability, or slower growth rates from unstable modes upstream of separation.

3.4 Sensitivity of Forcing Frequency and Location to α

α sweeps for fixed $\text{St}_e = \text{St}_e^*$ were conducted and comparisons of L/D to the baseline and suction surface with holes are shown in figures 24 and 25 so the α parameter appears as a local twist in wing profile. Comparisons with the baseline show improvements over a range of α with peaks in $\Delta L/D$, shown in dark red near α_{crit}. Improvements in $\Delta L/D$ persist past α_{crit}, but typically drop off quickly as α increases. The x_e^* locations identified from figure 23 show

Figure 24: Contours of $\Delta L/D$ relative to baseline from α sweeps at different forcing locations x_e. Black dashed lines denote α_{crit}. (+) indicates x_s at α_{crit}. Re $= 4, 6, 8 \times 10^4$ left to right.

Figure 25: Contours of $\Delta L/D$ relative to suction surface with x_{holes}, from α sweeps at different forcing locations x_e. Plotting conventions as previous figure.

improvements in $\Delta L/D$ which persists over the largest range of α. Light blue areas of decrease in $\Delta L/D$ occur in all three plots just above α_{crit} and downstream of x_s. Figure 25 has thinner α bands of L/D improvement compared with those in figure 24. The contour shapes are similar to those of figure 24 with the exception of Re $= 8 \times 10^4$, which changes from a contour with a maximum improvement near $x_e/c = 0.3$ at α_{crit} to two peaks at $x_e/c = 0.1$ and 0.5 also at α_{crit}. These changes in contour shapes can be attributed to the surface impact of holes causing some improvement in L/D, but the acoustic forcing significantly increases L/D beyond the improvements from increased surface roughness alone.

Changes in ensemble averaged flow fields due to the surface impact of holes and forcing at $x/c = 0.3$ can be observed in figure 26. The unforced flow fields with the smooth suction surface, in the left column of the figure, show separated regions originating near mid-chord and extending to the trailing edge. As Re increases and consequently α_{crit} decreases from top to bottom, the separation region thins and moves downstream. Introducing the speaker holes, shown by the contours in the middle column, has little effect on the Re $= 4 \times 10^4$ case (top), but collapses the separated regions for Re $= 6 \& 8 \times 10^4$ (middle & bottom). Similar to the switch in flow state post-α_{crit}, this change in the flow increases streamline curvature over the chord, increasing lift, and shrinks the cross-sectional area seen by the flow, decreasing drag. Finally, forcing at $x_e/c = 0.3$ collapses the Re $= 4 \times 10^4$ as well and shrinks the separated regions

Figure 26: Ensemble averaged PIV u velocity fields with streamlines for unforced one–piece model, unforced two-piece model with speaker holes at $x_{holes}/c = 0.3$, and forced two-piece model with speaker holes at $x_e/c = 0.3$ (Left to Right) at α_{crit} and $Re = 4, 6, 8 \times 10^4$ (Top to Bottom).

for $Re = 6 \,\&\, 8 \times 10^4$, relative to the case with holes and no forcing. The shrinking of the LSB leads to the increase L/D shown in figure 25. These changes in streamline curvature above the chord and streamwise cross-sectional area experienced by the flow explain the plateau and bowl shapes of C_L and C_D shown in figure 19.

3.5   Forcing at $Re = 2 \times 10^4$ Revisited

Further experiments were conducted to investigate the feasibility of effective forcing at $Re = 2 \times 10^4$. These experiments were conducted with a higher spanwise density of speaker locations ($z_e/b = 0.025 - 0.975$ by $z_e/b = 0.025$) at one chordwise position, $x_e/c = 0.1$. This quadrupling of linear speaker density resulted in 39 speakers, compared with the 9 of the previous wing. Figure 27 compares three separate frequency sweeps, each conducted at α_{crit}, as well as their average, with a similar sweep using the 9 speaker wing. Though the error bars are large, in part due to the difficulties in measuring the smaller forces characteristic of this lower Reynolds number, there is a consistent increase in L/D, from a baseline 7 to a maximum of 10. The plateau shape is in line with similar measurements on the 9 speaker wing at higher Reynolds numbers, as in figure 15

The large variance in the individual runs is a measure of the unsteadiness in the flow (in addition to single measurement uncertainty). The increased unsteadiness comes from the vortex formation, convection and shedding over the chord length and from long-period switching between alternate states I and II. Finally, the plateau of the averaged trace (right) has a maximum at $St_e^* = 4.0$, or $St_e^*/Re^{1/2} = 0.028$, similar to the 0.027 value found above.

Figure 27: (Left) three individual frequency sweeps at α_{crit} with $x_e/c = 0.1$ and 39 uniformly spaced spanwise speakers, and one frequency sweep with 9. (Right) same as left, but with the three sweeps averaged. The error bars at left show the standard deviations of each individual time series, while on the right, for the 39 speakers, they show run-to-run repeatability (the standard deviation of means). The $\mathrm{St}_e = 0$ data point corresponds to the unforced baseline at the beginning of each frequency sweep.

For the 9 speaker runs, the original flow response at Re $= 2 \times 10^4$ was weak, likely because the effective forcing amplitudes were low (see figure 16). The stronger response in L/D with roughly four times the number of speakers is due to a combination of increased spanwise coherence of the forcing, together with the higher local excitation amplitudes resulting from greater constructive behavior between discrete stations.

When forced at St_e^*, C_L increases over the range $7° \leq \alpha \leq 9°$ in contrast to the lack of measurable increase with 9 speakers. Similar tests were conducted for the higher Re (figure 28). For $\alpha > \alpha_{\mathrm{crit}}$, forcing yields no increase in C_L. Similarly, for angles where the 9 speakers already induce a complete switch to SII [Yang and Spedding, 2014], increased speaker density gives no additional performance benefit. By contrast, there are lower angles (i.e. $\alpha \in [0, 1]$ for Re $= 6 \times 10^4$) where the 9 speakers caused a partial switch to SII, while the 39 speakers induce a complete switch. Further, the 39–speaker traces show increased C_L for α where the 9 speakers were ineffective. Interpreting these results in light of the sigmoid-shape C_L vs excitation amplitude curves of Yarusevych et al. [2007], it can be concluded that varying number of speakers affects the flow in a similar manner to that of varying excitation amplitude, and that as Re or α decreases, the critical excitation amplitude increases.

Figure 28: Comparison of C_L vs α for smooth baseline, 9-speaker and 39-speaker wings, forced at St*. Re = $2, 4, 6, 8 \times 10^3$ from top left to bottom right.

4 Discussion and Conclusions

The aerodynamic performance of a wing in the transitional regime (Re $\in \{10^4, 10^5\}$] has a sensitive dependence on geometry and flow conditions. Small changes in flow speeds and angles of incidence result in the forming and bursting of LSBs with correspondingly abrupt changes in lift and drag. These sensitivities are driven by the easily–destabilized shear layer created by laminar separation, and are persistent over an order of magnitude range in Re. As Re increases, the boundary layer thins and viscous effects become less influential leading to an increase in C_L as the inviscid limit is approached. Similarly, for increasing Re, there is a decrease in C_D and α_{crit}, as the flow switches states from laminar separation without time-averaged reattachment to an LSB where the vortices arising from the shear-layer instability act to bring high-speed fluid down once again to the airfoil surface.

The baseline experiments acquired here and in previous studies [Yang and Spedding, 2014, Michelis et al., 2017, Tank et al., 2021, Toppings and Yarusevych, 2024] highlight the sensitivity of transitional flows to small perturbations, and the purpose of this paper is to show the viability of onboard sinusoidal acoustic forcing on a finite wall-bounded wing, with the objective of identifying favorable forcing frequencies and locations across a range of Re and α.

Experiments demonstrated significant improvement in aerodynamic performance with clear trends for optimal forcing parameters. Systematic variation of forcing frequencies (St_e) showed concave curves allowing for identification of optimal forcing frequencies

$$St_e^* = \underset{St_e}{\mathrm{argmax}} \frac{L}{D} \tag{4}$$

which collapsed across Re when scaled by $Re^{1/2}$ [Zaman and McKinzie, 1991]. The most amplified frequencies are related to a shear layer instability, similar to a Kelvin-Helmholtz instability [Watmuff, 1999, Diwan and Ramesh, 2009, Yarusevych et al., 2009], but seeded by disturbances originating upstream in the boundary layer. If a flat plate momentum thickness, θ_{FP}, is estimated just upstream of the separation location for a lengthscale, then the optimal forcing frequency can be predicted as for an inviscid shear layer instability [Ho and Huerre, 1984].

Sweeps through the chordwise forcing location, x_e showed the most effective forcing frequency was always at St_e^*, and that forcing was most effective upstream of separation location, x_s. Improvements in L/D diminish quickly downstream of separation, and the upstream distance between optimal forcing, x_e^*, and separation, x_s, locations increases with Re. Further work is necessary to disentangle the roles that forcing amplitude, instability growth rates, and instability onset location play in this last result.

In the lower Re range, the shear layer is more robust to surface imperfections, but targeted forcing still causes the separated region on the aft portion of the airfoil to collapse. At Re $\geq 6 \times 10^4$, surface roughness introduced by the speaker holes themselves was sufficient to collapse— at least partially—the separated region, passively improving the airfoil performance. Nevertheless, even with LSBs already formed on the airfoil surface, the additional acoustic forcing further improves airfoil performance by reducing the streamwise and wall-normal dimensions of the LSB.

In a uniform flow, far from any boundaries and for a given geometry, the normal and streamwise components of the aerodynamic force depend only on Re, and their ratio, L/D, is a convenient measure of performance. For a fixed airfoil shape, the problem geometry is varied through the angle of attack, α, so one expects

$$L/D = f(\mathrm{Re}, \alpha). \tag{5}$$

In practice, and especially for Re $\leq 10^6$, L/D is very sensitive to small departures from these ideal conditions, and ambient turbulence intensity, surface roughness (skin friction, C_f), and acoustic environment ($a(A_e, f_e)$), all can modify the flow state, as summarized in eq. (6).

$$L/D = f(\mathrm{Re}, \alpha, \underbrace{T, C_f, a(f_e, A_e) \ldots}_{\epsilon}) \tag{6}$$

$$\epsilon \approx F(f_e, A_e, \vec{x}_e) \tag{7}$$

One may think of these departures from ideal flow conditions as perturbations to the base flow set by Re and α, and setup a persistent forcing equivalence (eq. (7)) which will depend on the excitation frequency (f_e), amplitude (A_e), and location ($\vec{x}_e$).

Frequency sweeps of transitional flows by Zaman and McKinzie [1991] showed ΔC_L increases of up to 30% with a plateau around some preferred frequency. The plateaus collapsed with $St_e/Re^{1/2}$ scaling to $St_e/Re^{1/2} = 0.02 - 0.03$

depending on the airfoil. This agrees with the frequency collapse of maximum gains from Rolandi et al. [2025] for resolvent analysis across different Re on a NACA0012. The scaling has been extended here to show $(L/D)_{\max}$ collapsed to $\mathrm{St}_e/\mathrm{Re}^{1/2} = 0.027$ for the wall-bounded NACA 65(1)412. Expressing this as a single shear layer constant $C_{SL} \approx 0.025$, (owing to the forcing resonating with shear layer instabilities [Diwan and Ramesh, 2009, Rolandi et al., 2025]), it is possible to solve for the optimal forcing frequency, f_e^*,

$$C_{SL} = \frac{\mathrm{St}_e^*}{\sqrt{\mathrm{Re}}} = \frac{f_e^* \sqrt{c\nu}}{U^{3/2}} \Rightarrow f_e^* = U^{3/2} \frac{C_{SL}}{\sqrt{c\nu}}. \tag{8}$$

From eq. (8), f_e^* for a given airfoil will only be a function of the thermodynamic state and the freestream velocity. For many engineering flows ν can be assumed to be constant allowing for the optimal forcing frequency to be set by a single measurement of U.

Amplitude variation with forcing at the most unstable shear layer frequency for transitional flows has been explored by Yarusevych et al. [2007] for an airfoil and Michelis et al. [2017] for a flat plate with an imposed non-uniform pressure gradient. The combined measurements demonstrated that increasing the amplitude above some minimum threshold increases L/D for the airfoil and shrinks the separation bubble for the flat plate. Continued increase of the amplitude further increases L/D and shrinks the separation bubble until L/D saturates and the separation bubble is eliminated (at least up to the measurement resolution). This suggests that $(L/D)_{\max}$ is reached at an optimal amplitude, A_e^*, equal to the saturation amplitude A_{sat}.

The most favorable chordwise excitation location for forcing of transitional separated flows has consistently been shown to be at or upstream of the separation location, x_s [Hsiao et al., 1990, Greenblatt and Wygnanski, 2000, Yeh and Taira, 2019, Yeh et al., 2020]. This is in agreement with linear stability analysis of transitional flows which finds the onset of instabilities at or shortly after the onset of the adverse pressure gradient and upstream of separation [Diwan and Ramesh, 2009, Michelis et al., 2017, Yeh et al., 2020]. A naive assumption may be the optimal chordwise forcing location is at the onset of instability, as this is where exponential growth of perturbations will occur, but Yeh et al. [2020] found with resolvent analysis, that slightly downstream was optimal for a forcing mode magnitude cost function. Likely, determining a chordwise optimal location will depend on the target range of α (which dictates the onset of the adverse pressure gradient and consequently flow instability and separation), and the trade-offs between input energy and performance gain. Here, it has been shown that as long as $x_e \leq x_s$ the bulk of L/D improvement is achieved. From an L/D perspective, as long as the forcing is able to introduce a perturbation at $A_e^* = A_{\mathrm{sat}}$ and $\mathrm{St}_e^* = C_{SL}\mathrm{Re}^{1/2}$ downstream of instability onset, then L/D will be maximized. This implies that suboptimal actuator placement can be mitigated by A_e modification as long as $x_e \leq x_s$. Yang and Spedding [2014] provide some evidence for this through consistent improvement in $\Delta\%L/D$ regardless of x_e/c as long as $\mathrm{St}_e/\mathrm{Re}^{1/2} \approx 0.03$.

Literature exploring forcing simultaneously at multiple chordwise locations is limited. Lyon [1997] used trips at multiple locations in an effort to reduce LSB drag finding that tripping the flow at a single location always yielded equal or better drag reduction compared with trips at multiple chordwise locations. Yeh and Taira [2019] examined global forcing vs local forcing at a single chordwise location on a NACA0012 and found that the input-output gain was reduced by an order of magnitude through the restriction of forcing to a 2-D surface at a single chordwise location which spanned the wing surface. Some of this reduction can be attributed to the restriction to a 2-D surface and it should be noted even for the global forcing, the optimal inputs take the form of a continuous chordwise region near the leading edge with a length $< 0.05c$.

Optimal spanwise distribution of forcing is still an open question, although recent experimental and numerical studies have begun to explore spanwise structure [Michelis, 2017, Yeh and Taira, 2019, Yeh et al., 2020, Toppings and Yarusevych, 2024, Rolandi et al., 2025, Klose et al., 2025]. Michelis [2017] demonstrated the development of spanwise modulation of the coherent rollers in an LSB through selected POD modes, and Yeh and Taira [2019], Rolandi et al. [2025] showed varying speeds of transition and flow reattachment depending on spanwise modes and commensurate forcing in a resolvent-based analysis. Rolandi et al. [2025] performed a biglobal resolvent analysis to investigate the role of spanwise modes on a NACA0012 airfoil at for $\mathrm{Re} \in \{2500, 10000\}$, and distinguished between the influence of shear-layer and wake modes at short and long timescales, respectively. Here, the forcing is continuous in time (as opposed to impulsive [Michelis et al., 2017]), so a mix of shear-layer and wake modes may be expected, as shown in some detail in Klose et al. [2025]. The relative importance of three-dimensional modes can also be affected by compressibility (sometimes used to stabilise numerical solutions) [Miotto et al., 2022] and by proximity to the wall of the main shear layer modes [Liu et al., 2023].

Though the interplay of boundary layer, shear layer and wake instabilities yield a wide range of paths to closed laminar separation bubbles and their stability, the findings reported here are general to wall-bounded transitional flows with applications that extended to both higher and lower chord-based Re as shown by Yeh and Taira [2019], Coskun et al.

[2024] and Ribeiro and Taira [2024], and boundary layer flows with adverse pressure gradients [Michelis et al., 2017, Celik et al., 2023].

The results described here suggest that significant improvement in L/D across a range of Re, α, and different airfoil contours is possible through the introduction of spanwise acoustic excitation at a single chordwise location near the leading edge with most effective frequencies near a normalised value of $St_e/Re^{1/2} = 0.027$.

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